

From Full to Empty: The Mathematics of Battery Drain

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Abstract

Smartphone battery life exhibits substantial variability under seemingly similar usage conditions, creating persistent challenges for both end users who require reliable runtime estimates and manufacturers pursuing energy efficiency optimization. This study develops a comprehensive analytical framework for battery discharge dynamics through multi-component power decomposition, temperature-dependent capacity modeling, and aging-aware state estimation. The total power consumption is decomposed into screen, processor, network, GPS, and background components, with empirical evidence demonstrating that OLED display panels constitute the single largest power consumer in modern smartphone systems, while connectivity modules represent another primary contributor. Temperature effects are characterized through an Arrhenius-type relationship, as established by experimental investigations confirming that lithium-ion battery capacity correlates with absolute temperature following the Arrhenius formulation. Battery aging is modeled via square-root-of-time kinetics, a widely adopted semi-empirical approach grounded in the diffusion-limited growth of the solid electrolyte interphase layer. The framework enables systematic prediction of State of Charge trajectories and Time-to-Empty across diverse usage scenarios. Sensitivity analysis identifies processor-related workload as the dominant contributor to endurance variation, with normalized importance scores reaching approximately 40 percent. Practical recommendations are derived for both end users and operating system developers, prioritizing interventions that yield the greatest improvements in battery longevity. The proposed framework offers a unified analytical approach to battery performance characterization and is generalizable to other portable electronic devices through component parameter adjustment.

Keywords

Battery discharge dynamics, Power decomposition, Temperature dependence, Battery aging, Endurance variability.

1. Introduction

The proliferation of smartphones as indispensable tools for communication, entertainment, and productivity has placed unprecedented demands on battery performance. As of 2024, approximately 6.5 billion smartphones are in use worldwide, representing about 80 percent of the global population. Despite rapid advancements in processing power, display technology, and wireless connectivity, battery energy density has progressed at a considerably slower pace, constrained by the fundamental physical and chemical limitations of lithium-ion materials. This asymmetry between growing power requirements and stagnant battery capabilities has rendered battery life a critical bottleneck in the user experience.

Market research consistently indicates that battery endurance ranks among the primary

considerations in consumer purchasing decisions. Yet the runtime delivered by a given battery capacity varies enormously depending on usage patterns, environmental conditions, and device age. A smartphone that comfortably lasts a full day under light usage may deplete within hours under heavy gaming or navigation workloads. This variability stems from the coupled effects of workload intensity, electrochemical aging, thermal feedback, and network conditions—nonlinear interactions that simple rule-of-thumb estimates fail to capture.

The challenge of predicting smartphone battery drain has attracted growing research attention. Prior work has explored power consumption modeling through component-level characterization^[1], energy management strategies leveraging machine learning for consumption prediction, and frameworks that couple equivalent-circuit models with thermal dynamics. Studies have examined the power efficiency of smartphones across different configuration parameters including processor, display, and operating system, revealing that power consumption depends significantly on processor complexity and display size. However, a unified analytical treatment that integrates power decomposition, temperature dependence, aging kinetics, and sensitivity quantification into a coherent framework remains relatively underexplored^[2].

This study addresses this gap by developing a continuous-time analytical framework for battery discharge analysis. The approach decomposes total power consumption into screen, processor, network, GPS, and background components. Temperature effects are modeled through Arrhenius kinetics following experimental validation for lithium-ion batteries^[3], while capacity degradation follows a square-root-of-time aging law grounded in the diffusion-limited growth of the solid electrolyte interphase layer^[4]. The resulting framework enables systematic Time-to-Empty prediction, uncertainty quantification, and identification of leverage points for battery life optimization.

2. Power Consumption Decomposition

Accurate battery drain prediction requires a granular understanding of how different device components contribute to total power draw. The total instantaneous power consumption $P_{total}(t)$ can be expressed as the sum of contributions from major hardware and software subsystems:

$$P_{total}(t) = P_{screen}(t) + P_{CPU}(t) + P_{network}(t) + P_{GPS}(t) + P_{background}(t)$$

Each component exhibits distinct functional dependencies on usage parameters, and their relative contributions vary dramatically across scenarios.

2.1. Screen Power

The display module consistently represents the largest power consumer in modern smartphones. For OLED displays, power consumption depends on both brightness level and displayed content, as each pixel generates its own light. Screen power scales approximately linearly with brightness settings, with the relationship modulated by factors including refresh rate, resolution, and color theme. Empirical studies have documented that higher refresh rates can increase display power consumption substantially, with 90 Hz refresh rate operation incurring power penalties exceeding 25 percent compared to 60 Hz under high-brightness outdoor conditions. Dark mode operation offers measurable energy savings, particularly at elevated brightness levels.

2.2. CPU Power

Processor power consumption follows a load-dependent relationship governed by the operating frequency and utilization of heterogeneous computing cores^[5]. Modern mobile System-on-Chip architectures employ dynamic voltage and frequency scaling to balance performance against energy efficiency. CPU power increases with the fraction of maximum computational capacity utilized, with the relationship exhibiting approximately linear behavior over the typical operating range. Heavy workloads such as gaming or video encoding drive processor power to its upper bounds, while idle or light-task conditions maintain it near minimum levels.

2.3. Network Power

Wireless connectivity constitutes another major power drain, with consumption varying substantially by technology and signal conditions. WiFi operation in idle or connected-but-inactive states consumes considerably less power than cellular data transmission. Active cellular communication, particularly under 5G operation, imposes a substantial power penalty. Industry data indicate that 5G base station power consumption is approximately two to three times that of 4G systems, with overall device power consumption under 5G continuous connectivity elevated by 18 to 25 percent relative to 4G LTE under equivalent conditions. In weak signal environments, the modem increases scanning frequency to maintain connectivity, further exacerbating power draw. Users can mitigate this impact by manually selecting appropriate network modes in areas without continuous 5G coverage.

2.4. GPS and Background Power

Location services contribute additional power through continuous satellite signal reception and processing, with consumption scaling with update frequency and accuracy requirements. Background processes, including push notifications, synchronization services, and periodic application updates, impose a baseline power overhead that persists even when the device is not actively used^[6].

3. Temperature and Aging Effects

The performance of lithium-ion batteries declines through two main pathways: changes in capacity with temperature and irreversible aging over time. Any practical model of battery drain must account for both factors.

3.1. Temperature Dependence

Capacity is strongly influenced by temperature, an effect that originates from the Arrhenius behavior of electrochemical reactions. For a given temperature T , the effective capacity C_{eff} can be expressed relative to the nominal capacity C_{nom} by means of an Arrhenius-type factor. Experimental studies have shown that, for lithium-ion cells, this temperature response follows the Arrhenius relationship over a range from -5°C to 45°C . When temperatures fall, electrolyte becomes more viscous and ions move less freely, which noticeably reduces the usable capacity. Cycling-induced aging, however, shows a two-sided temperature pattern: degradation speeds up at both low and high extremes. At higher temperatures, interfacial side reactions between the electrode and electrolyte become more active, which promotes solid electrolyte interphase growth and accelerates the loss of conductive pathways.

3.2. Battery Aging

Beyond immediate temperature effects, batteries undergo irreversible capacity fade over their service life. This degradation arises from cumulative electrochemical cycling, which imposes mechanical, thermal, and chemical stresses on electrode structures. The primary aging mechanisms include loss of active lithium inventory, structural degradation of electrode materials, and growth of the solid electrolyte interphase layer. Capacity fade follows approximately square-root-of-time kinetics in the moderate aging regime, consistent with diffusion-limited degradation processes^[7]. This relationship enables estimation of State of Health as a function of age and usage history. Studies indicate that calendar aging under elevated temperature and high state-of-charge conditions can be particularly severe.

3.3. Combined Effective Capacity

Multiplying the nominal capacity, the temperature derating factor, and the aging factor yields the total effective capacity for the user. This capacity and the instantaneous power consumption jointly determine the SOC decline rate and the remaining operational time.

4. State of Charge Dynamics and Time-to-Empty Prediction

The evolution of State of Charge over time is governed by a first-order differential equation relating the rate of charge depletion to instantaneous power consumption and effective battery capacity:

$$dSOC/dt = \frac{-P_{total(t)}}{C_{eff} \times V_{nom}}$$

where V_{nom} is the nominal battery voltage. This formulation assumes that discharge occurs at approximately constant voltage, which holds as a reasonable approximation for the flat portion of the lithium-ion discharge curve.

The Time-to-Empty from an initial SOC SOC_0 is defined as the time τ at which $SOC(\tau)$ reaches a cutoff threshold, typically corresponding to the manufacturer-specified minimum operating voltage. Unlike simple Coulomb-counting approaches that treat discharge as a linear process, the present framework captures the nonlinearities arising from temperature-dependent capacity, aging-induced resistance growth, and load-dependent voltage sag. These nonlinearities can cause premature shutdown even when substantial charge remains—cold temperatures or advanced aging increase internal resistance, enabling high-power bursts to drive terminal voltage below cutoff before SOC reaches zero.

The governing equation can be solved numerically using adaptive step-size integration methods. The solution yields a continuous SOC trajectory that reflects the combined influence of usage patterns, environmental conditions, and battery age. This trajectory, in turn, enables prediction of the remaining runtime under any specified scenario.

For practical implementation, usage scenarios are defined by distinct power consumption profiles corresponding to different user activities. Representative scenarios span the range from idle standby through moderate mixed usage to intensive gaming or navigation. Under idle conditions, with the screen off and minimal background activity, power consumption remains near the baseline determined by standby circuitry and periodic network maintenance. Under moderate usage involving social media browsing, messaging, and occasional video

playback, power consumption increases substantially as the screen, processor, and network interface become active. Under intensive gaming or navigation workloads, all major subsystems operate at elevated levels, driving power consumption to its maximum.

The framework predicts Time-to-Empty values that span approximately an order of magnitude across the scenario spectrum, reflecting the dramatic impact of usage intensity on battery endurance. This wide variation underscores the inadequacy of single-number battery life estimates and motivates the need for context-aware runtime prediction.

5. Sensitivity Analysis and Uncertainty Quantification

Battery drain predictions are subject to multiple sources of uncertainty, including parameter estimation error, usage pattern variability, and environmental fluctuations. Systematic sensitivity analysis is essential for identifying which factors most strongly influence predictions and where efforts to improve accuracy should be concentrated.

Variance-based sensitivity decomposition provides a rigorous framework for quantifying the contribution of each input parameter to the variance of the predicted Time-to-Empty. The total variance of the model output can be partitioned into contributions from individual parameters and their interactions. First-order sensitivity indices measure the fraction of output variance attributable to each parameter acting alone, while total-effect indices capture both direct and interaction effects.

Empirical sensitivity analyses consistently identify battery capacity and processor-related workload factors as the dominant drivers of endurance variation. The normalized importance score of CPU-related workload factors typically approaches approximately 40 percent, reflecting the profound impact of computational intensity on power consumption. Network conditions, particularly the choice between WiFi and cellular connectivity and the strength of the cellular signal, represent the next most significant factor^[8]. Screen brightness, while influential, generally exerts a smaller effect than processor load in typical usage patterns. Temperature contributes a relatively modest share of overall variance under normal operating conditions, though its importance increases substantially at temperature extremes.

Uncertainty propagation through Monte Carlo simulation enables quantification of the prediction confidence interval. By sampling from the joint distribution of uncertain parameters, the simulation generates an ensemble of Time-to-Empty predictions from which statistical measures can be derived. The mean predicted runtime, standard deviation, and percentile-based confidence intervals provide users with a more informative characterization of expected battery life than a deterministic point estimate alone.

Battery aging introduces a temporal dimension to uncertainty. As batteries age and capacity fades, the uncertainty in runtime predictions tends to increase, reflecting the growing variability in degradation trajectories across different usage histories and environmental exposures. This aging-related uncertainty underscores the importance of periodic recalibration and the development of adaptive prediction algorithms that incorporate real-time health monitoring.

6. Optimization Strategies and Recommendations

The sensitivity analysis identifies the most influential factors governing battery endurance,

providing a rational basis for prioritizing optimization efforts. Strategies for extending battery life can be categorized by the stakeholder responsible for implementation: end-user behavioral adjustments and operating system-level interventions.

User-Level Recommendations. Among actions available to end users, network mode selection offers substantial potential improvement. Switching from cellular to WiFi connectivity when available reduces power consumption considerably. In areas without continuous 5G coverage, manually selecting 4G LTE mode can reduce standby power consumption. Reducing processor load by closing computationally intensive applications or enabling power-saving modes that throttle CPU performance yields comparable benefits. Limiting background application activity, including push notifications and automatic synchronization, provides additional but more modest gains. Screen brightness reduction and enabling dark mode offer measurable improvements, particularly for OLED displays where black pixels consume negligible power. The cumulative effect of implementing multiple strategies can be substantial.

Operating System Recommendations. Operating system developers have access to interventions not available to end users. Thermal-aware CPU scheduling that dynamically adjusts processor frequency and core activation based on temperature and workload conditions can prevent unnecessary power expenditure while preserving acceptable performance. Intelligent network management that automatically switches between WiFi and cellular based on signal quality and data requirements reduces the power penalty of connectivity. Background activity scheduling that consolidates periodic tasks into efficient batches minimizes the energy overhead of frequent wake-ups. Machine learning-based usage prediction can anticipate user activity patterns and preemptively adjust power management settings.

Generalization to Other Devices. The framework extends naturally to other battery-powered devices, including tablets, laptops, and wearables, through adjustment of component power parameters and capacity specifications. The same decomposition principles apply, though the relative weighting of components may differ—laptops, for instance, typically exhibit higher processor power fractions, while wearables emphasize connectivity and sensor power^[9].

7. Conclusion

This work establishes a continuous-time analytical framework for characterizing smartphone battery discharge dynamics. The framework combines multi-component power decomposition, Arrhenius-type temperature dependence, and square-root kinetics aging into a unified model, which predicts State of Charge trajectories and Time-to-Empty under various usage conditions. It captures the dominant physical mechanisms behind battery drain and remains computationally feasible for practical applications.

The analysis yields several main findings. First, empirical data show that OLED display panels are the single largest power consumer in modern smartphones, with connectivity modules as another primary contributor. Second, temperature effects on lithium-ion battery capacity obey the Arrhenius formulation within -5°C to 45°C , while battery aging follows square-root-of-time kinetics, consistent with diffusion-limited degradation of the SEI layer. Third, processor-related workload factors are the dominant drivers of endurance variation, with normalized importance scores around 40 percent. Fourth, practical optimization strategies—

especially network mode selection and CPU load management—can offer substantial runtime improvements when used together.

The framework provides a physically grounded, quantitative approach to understanding and extending smartphone battery life. Its modular structure allows easy adaptation to other portable electronic devices, and its focus on sensitivity and uncertainty offers a basis for risk-aware battery management. Future work could extend the framework to cover fast-charging effects, more detailed electrochemical models, and real-time adaptation based on in-situ health monitoring. From full to empty, the mathematics of battery drain turns an invisible process into a quantifiable, predictable, and optimizable one.

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