ISSN:2790-1688 Volume-15-(2025)

Improved NSGA-II for Multi-Objective Optimization of Acoustic Array GDOP under Physical Constraints

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Abstract. To address the issue of restricted array layout in wide-area acoustic monitoring, an improved NSGA-II multi-objective optimization algorithm that integrates the feasible region hard constraint initialization (FDHCI) strategy is proposed. This algorithm aims to minimize the geometric accuracy factor (GDOP) and minimize the violation degree of constraints, with two objectives of minimizing GDOP and performing the lowest violation degree. Through the rejection sampling mechanism of the FDHCI strategy, the algorithm ensures that the initial population completely falls within the feasible region, avoiding the waste of resources in repairing infeasible solutions in the early stage. In a monitoring scenario of 200×180m with 4 building obstacles, 32 sensors were deployed as FDHCI-optimized array, basic NSGA-II-optimized array, and traditional-optimized uniform array. The results show that the initial violation degree of constraints of the FDHCI-optimized array (10^0) is 6 orders of magnitude less than that of the basic NSGA-II (10^6); the proportion of high-precision positioning area is more than that of the basic NSGA-II and traditional optimized arrays by 4.2% and 4.4% points respectively. The study indicates that constraint compliance and positioning accuracy of the array is optimized significantly by the FDHCI strategy which improving the feasibility of the initial population and the evolutionary efficiency, providing data support for the performance improvement of the acoustic TDOA positioning system.

Keywords: Constrained acoustic array configuration; wide-area acoustic monitoring; FDHCI; NSGA-II; GDOP.

1. Introduction

Wide-area acoustic monitoring [1] plays a crucial role in various application scenarios such as environmental monitoring, security systems, and tracking wildlife. Time Difference of Arrival (TDOA) time delay positioning is one of the mean technologies for sound source localization [2]. It calculates the sound source position by detecting the time difference of signal arrival at different array elements. The positioning accuracy fundamentally depends on the collaborative optimization of the array element geometry and time delay estimation. In this process, the Geometric Dilution of Precision (GDOP) directly quantifies the amplification effect of sensor spatial distribution on the uncertainty of the positioning result, which is a core indicator for measuring and predicting system performance [3].For the optimization problem of array layout, existing research has laid an important foundation from theoretical optimal configuration analysis [4,5], evolutionary algorithm optimization [6], and exploration of the mechanism of positioning blind zones [7]. Most of these works are based on the assumption of an ideal test range area and unconstrained conditions, pursuing the global theoretical optimal solution of GDOP. However, in actual engineering deployment, the configuration of the optimal array is usually constrained strongly by multiple strict physical constraints, resulting the theoretical models could not be applied directly or easily.

The sensor layout is constrained by numerous such limiting conditions: Firstly, there are geographical and environmental restrictions, including areas that are undeployable due to obstacles. Secondly, acoustic effects require the layout to adapt to the terrain and avoid reflection by obstacles. Thirdly, there are cost constraints, with limits on the number and performance of nodes, and high-cost solutions being unfeasible. Fourthly, special man-made rules also restrict the location selection of the equipment.

Aiming to establish a systematic framework to automatically optimize sensor positions while strictly adhering to actual deployment constraints such as geography, cost, and rule, an intelligent station placement optimization method based on a constraint handling algorithm of NSGA-II is proposed, in order to minimize or improve the GDOP performance significantly within the monitoring area. To be compared with traditional station placement strategies that rely on engineering experience or simplified geometric rules, this method provides a more scientific, efficient, and reusable solution for array designing in wide-area acoustic monitoring systems under complex constraints, and is expected to enhance the positioning accuracy significantly and reliability of the system in practical scenarios.

2. GDOP and NSGA-II optimization theory

2.1 GDOP calculation model

Suppose there are N array elements (or multiple sub-array equivalent centers) in a two-dimensional plane with determined coordinates (x_i, y_i) ($i = 1, 2, \dots, N$), with an source's un-determined coordinates of $\mathrm{p} = (x, y)^T$, sound speed c. Assuming the initial element as the reference spot, and the TDOA estimated value of the i element relative to the reference element is:

$$\tau_i(\mathrm{p}) = \frac{1}{c}(\|p - p_i\| - \|p - p_1\|) + n_i \tag{1}$$

where $\mathrm{p}_i = (x_i, y_i)^T$ is the coordinate of the i-th element, and n_i is detected noise.

Each row of $G(p)$ corresponding to the i element, represents the partial derivative of the TDOA observation values with respect to the source position p is:

$$G_{i-1}(p) = \frac{\partial \tau_i(p)}{\partial p} = \frac{1}{c}\left(\frac{p-p_i}{\|p-p_i\|} - \frac{p-p_1}{\|p-p_1\|}\right) \tag{2}$$

This formula essentially represents difference of direction vectors pointing to the source by the elements. Under assumption that the detected noise is independently and identically distributed Gaussian, the FIM can be simplified as:

$$F(p) = \frac{1}{\sigma_\tau^2} G(p)^T G(p) \tag{3}$$

The GDOP is defined as square root of trace of covariance matrix of the position estimation error. Substituting the above covariance matrix gives:

$$GDOP = \sqrt{tr(cov(\hat{p}))} = \sqrt{tr(\sigma_\tau^2 (G^T G)^{-1})} \tag{4}$$

In engineering applications, it is usually assumed that $\sigma_\tau = 1$, simplifying to:

$$GDOP = \sqrt{tr((G^T G)^{-1})} \tag{5}$$

2.2 GDOP Optimization with Constraints Based on NSGA-II

The second-generation non-dominated sorting genetic algorithm (Nondominated Sorting Genetic Algorithm II, NSGA-II) was proposed by Deb et al. as a significant and widely used multi-objective evolutionary algorithm (MOEA) to avoid the limitations of its predecessor algorithms [8]. This algorithm is specifically designed to find a set of Pareto-optimal solutions that represent the best trade-offs among multiple conflicting objectives in a single simulation run.

The GDOP optimization problem with layout constraints based on NSGA-II can be formulated as a two-objective constrained optimization problem, and the steps are as follows:

2.2.1 Decision variables

$$X = [(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)] \tag{6}$$

where, N represents the number of sensors, and (x_i, y_i) denotes the two-dimensional coordinates of the i-th sensor. A solution X represents a complete station placement plan.

2.2.2 The objective function to be minimized

$$\text{Minimize } \mathrm{F(X)} = [\mathrm{f_1(X)}, \mathrm{f_2(X)}]^{\mathrm{T}} \tag{7}$$

where, $f_1(\mathrm{X}) = \mathrm{GDOP_{avg}(X)}$: This station placement scheme's average geometric accuracy factor in the monitoring area, used to maximize the overall positioning accuracy, $f_2(\mathrm{X}) = \sum_{i=1}^{N} \mathrm{DistanceToFeasible(sensor}_i)$: The total constraint violation degree, which is the sum of the distances from all sensor coordinates to their nearest feasible points (if the sensor is within the feasible area, the distance is 0). Minimizing this value forces the solution to satisfy the physical constraints.

2.2.3 Constraints

$$\text{Subject to } X \in \Omega \tag{8}$$

where, Ω represents the complex feasible region defined by geographical boundaries, obstacles, etc. This constraint is a hard constraint and is handled through specific mechanisms in this algorithm.

2.2.4 Constraint dominance principle

This principle is the core of NSGA-II in handling constraints. For any two solutions X_1 and X_2, X_1 is said to dominate X_2 under constraints ($X_1 \prec_c X_2$), if and only if any of the following conditions are met: X_1 is a feasible solution $f_2(X_1) = 0$, while X_2 is an infeasible solution $f_2(X_2) > 0$.Both are infeasible, but X_1 has a smaller total violation degree $f_2(X_1) < f_2(X_2)$.Both are feasible, and X_1 dominates X_2 in the Pareto sense (that is, $\forall m, f_m(X_1) \leq f_m(X_2)$ and $\exists m, f_m(X_1) < f_m(X_2)$.

3. Simulation Modeling

3.1 Simulation environment and parameter settings

Aiming to systematically investigate the effectiveness of the NSGA-II-based acoustic array layout optimization algorithm under complex constraints, a simulation experimental system is established as follows. To simulate the real wide-area acoustic monitoring scenario, a certain building complex is selected as an example for approximated modeling, and a complex simulation environment containing multiple obstacles is constructed, along with the reasonable configuration of algorithm parameters.

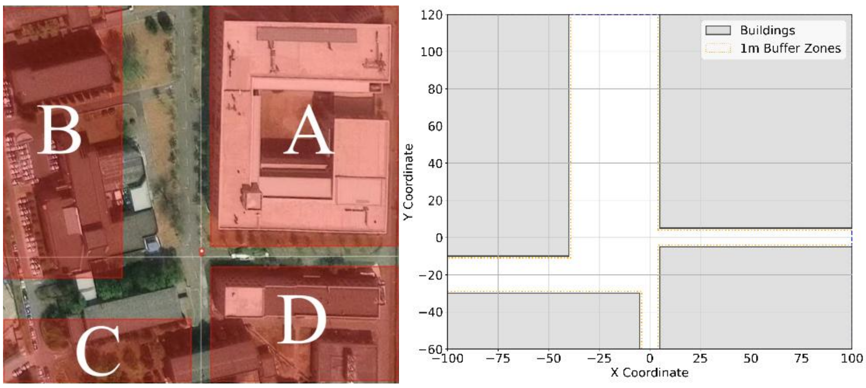

Fig. 1 (a)Satellite map of a building complex (b)Simplified model of obstacle areas

As shown in Figure 1, the monitoring area is defined as a rectangular area. The coordinates of the diagonal vertices in the horizontal direction range from -100 to 100 meters, and in the vertical direction, it ranges from -60 to 120 meters, with a total area of 200×180 square meters. There are four impassable rectangular building obstacles within this area, and their specific coordinate ranges are shown in Table 1. The four buildings are hard constraints. Sound reflection of these four buildings in 1-meter closing is assumed soft constraint area.

Table 1. Definition of obstacle coordinates

Building Number	X range	Y range
A	5 to 100	5 to 120
B	-100 to -40	-10 to 120
C	-100 to -5	-60 to -30
D	5 to 100	-60 to -5

3.2 TDOA performance evaluation indicators

To comprehensively evaluate the performance of different deployment schemes, this chapter defines two core evaluation indicators: the Average Geometric Dilution of Precision (Average GDOP) and the Constraint Violation.

3.2.1 Average Geometric Dilution of Precision

The Average GDOP is the core indicator for the global positioning accuracy of the acoustic array. For a given sensor layout, the geometric precision factor (GDOP) is calculated at all valid test points averagely. The smaller the value of this indicator is, the perfecter the average positioning accuracy of the sensor array in the monitoring area should be. The calculation formula is as follow:

$$\text{Average GDOP} = \frac{1}{M}\sum_{j=1}^{M}\sqrt{\text{tr}(\text{FIM}_j^{-1})} \tag{9}$$

where M represents the total number of valid test points.

3.2.2 Degree of Constraint Violation

The constraint violation degree is used to quantify the degree to which the sensor layout satisfies the physical constraints, with an ideal value of 0, that indicates complete compliance with all constraints. The calculation of this indicator includes two parts: hard constraints and soft constraints:

Hard constraint violation degree: If a sensor is located in any building (both the horizontal and vertical coordinates are in the building's range), the hard constraint violation degree of that sensor should be set to infinity, and then the corresponding layout scheme should be judged to be infeasible.

Soft constraint violation degree: For sensors located in a 1-meter buffer zone around the building, their soft constraint violation degree decreases linearly with the distance d face to the building's boundary: $\text{d}_i = max\,(0,1-d)$. This strategy guides the optimization algorithm to set sensors in areas far from buildings, to reduce the interference coursed by sound’s reflection.

Total average degree is the average of sum of all sensors’ constraint violation degree:

$$\text{Average Violation} = \frac{1}{N}\sum_{i=1}^{N}\left(v_{\text{hard},i} + v_{\text{soft},i}\right) \tag{10}$$

3.3 Feasibility Domain Hard Constraint Initialization

To enhance the performance of evolutionary algorithms in dealing with complex constrained optimization, a feasible domain hard constrained initialization strategy (FDHCI) is proposed. The underlying principle is to ensure all initial individuals to be strictly within the feasible domain of this issue during the population initialization stage through a systematic rejection sampling process. This provides a high-quality and fully feasible starting point for the subsequent evolutionary search.

For an optimization algorithm with m inequality constraints $g_i(x) \le 0$ and p equality constraints $h_j(x) = 0$, the violation degree of the constraints can be quantified as:

$$\text{cv}(x) = \sum_{i=1}^{m}\max(0, g_i(x)) + \sum_{j=1}^{p}|h_j(x)| \tag{11}$$

The goal of the FDHCI strategy is to construct an initial population $P_0 = \{x^{(1)}, x^{(2)}, \dots, x^{(N)}\}$, where each individual $x^{(i)}$ satisfies $\text{cv}(x^{(i)}) = 0$. The implementation process is as follows:

To generate a population of size N, for each individual to be initialized, the algorithm randomly selects candidate solution of x uniformly from the entire search space Ω and calculates its $\text{cv}(x)$; Only when $\text{cv}(x) = 0$, solution of x could be accepted as a member of the population, otherwise it is rejected and re-selected until a feasible solution is obtained. To ensure robustness of calculation, an upper limit on the maximum number of attempts is set for each individual; Fall mechanism should be activated while no feasible solution was obtained after reaching the limit, such as selecting the solution with a minimum violation degree from previous tried samples.

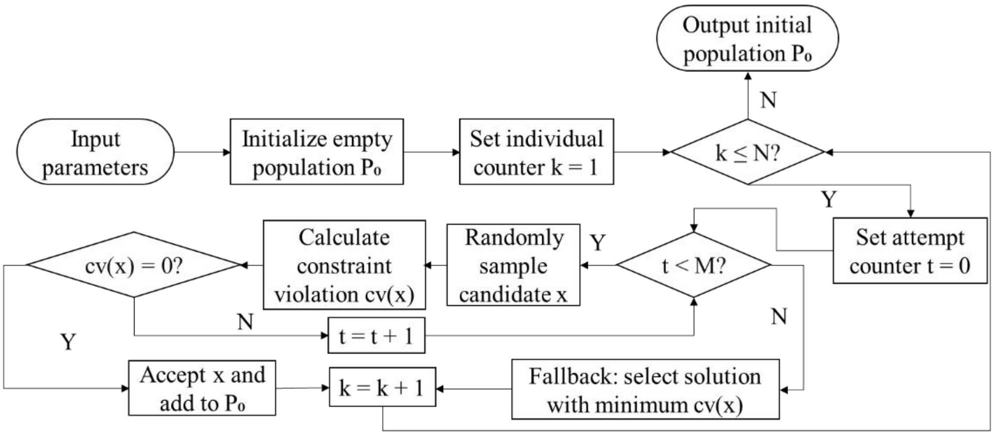

Fig. 2 Diagram of the FDHCI algorithm

4. Results and analysis

4.1 Convergence analysis of the optimization process

The total number of sensors is set to $N = 32$. The parameters for the NSGA-II algorithm are configured as: the population size is 100, and the maximum number of evolutionary generations is 100. Simulated binary crossover (SBX) is used with crossover probability $P_c = 0.9$ and distribution index $\eta_c = 20$; polynomial mutation is adopted with mutation probability $P_m = 1/64$ and distribution index $\eta_m = 20$. While the solution is infeasible, a repair operation could be activated, projecting their coordinates to the nearest feasible point.

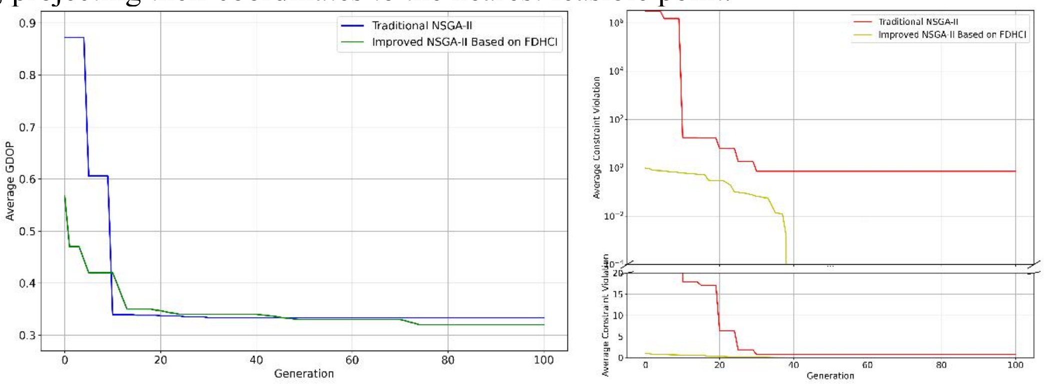

Fig. 3(a) Evolution curves of average GDOP (b) Evolution curves of average constraint violation

From Figure 3(a), the initial GDOP (0.567) of the improved FDHCI version is significantly better than that of the traditional algorithm (0.872), benefited by a better starting point for subsequent evolution. From Figure 3(b), The initial constraint violation degree of the traditional NSGA-II is as high as 10^6 orders of magnitude, while the improved version of FDHCI is only about 10^0. The gap is 6 orders of magnitude, which resource consumption of repairing infeasible solutions is avoided fundamentally in the early stage of the algorithm. The traditional algorithm stagnates at a pseudo-optimal state of 10^1 level after 10 generations, while the improved version of FDHCI converges to a constraint violation degree close to 0 in early 40 generations. Not only does it have a faster convergence speed, but it also avoids effectively the local optimal trap in the infeasible domain. Therefore, FDHCI, through strict initial feasibility guarantees, has a faster convergence rate and better robustness during iterations, improving the final solution quality of the algorithm.

4.2 Comparison analysis of three array layouts

4.2.1 Constraint Compliance of Array Layout

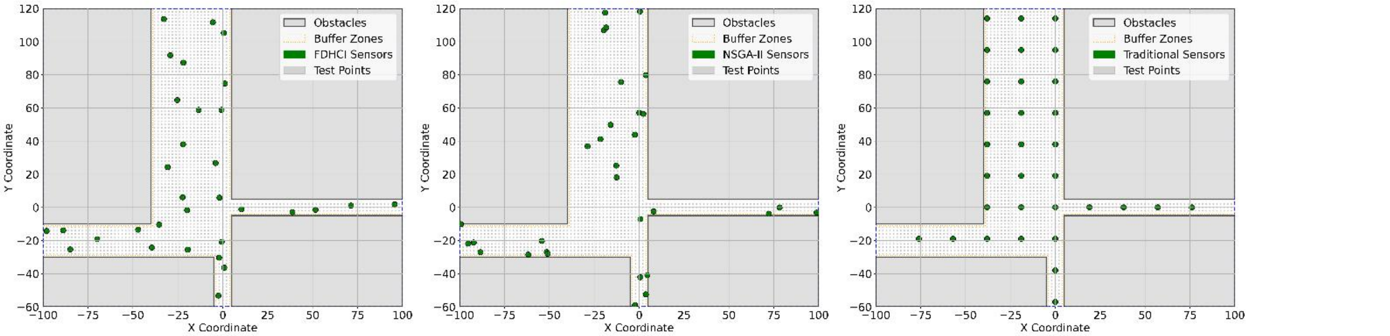

Fig. 4 (a) FDHCI optimized array (b) Basic NSGA-II optimized array (c) Traditional array

FDHCI Optimized Array (Figure 4 (a)):Sensors (green dots) are strictly distributed within the feasible region, completely avoiding all obstacles (gray area) and a 1-meter buffer zone (light gray area). Its distribution exhibits adaptive uniformity: evenly deployed in raw fields (such as the left and right sides, and the bottom), actively adjusting positions at the edges of obstacles to maintain coverage, without any sensor falling into the constraint area. This characteristic stems from the strict initial feasibility satisfies the FDHCI strategy. The initial population is ensured entirely feasible solutions by rejecting sampling. Subsequent evolution optimizes uniformity and coverage efficiency of the layout furtherly.

Basic NSGA-II Optimized Array (Figure 4 (b)):there are obvious faults in this sensor distribution: some sensors may close buffer zone (such as sensors on the right side of the middle obstacle), or close edges of obstacles, results in potential constraint violation. The overall layout is relatively concentrated, with insufficient coverage at the edges of the monitoring area (such as the left bottom). This phenomenon comes from the random initialization strategy of the basic NSGA-II, with the initial population containing a large number of infeasible solutions. Although repair operators by later adjustments may be activated, there even exists some unreasonable layout issues.

Traditional Optimized Uniform Array (Figure 4 (c)) : An uniform distribution with grid-point-control strategy is used to generate a traditional uniform array: take its origin of the grid point as reference, a uniform square grid is constructed.

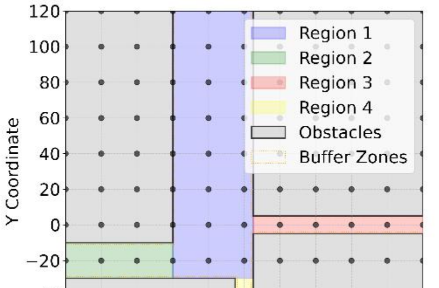

Fig. 5 Schematic of Traditional Uniformly Distributed Grid Point Control Strategy

As shown in Figure 5, by controlling the grid side length a, exactly 32 grid points fall within the four pre-defined feasible regions (Region 1: (-40, -30) ~ (5, 120); Region 2: (-100, -10) ~ (-40, -30); Region 3: (5, 5) ~ (100, -5); Region 4: (-5, -30) ~ (5, -60)), artificially avoiding constraint areas such as obstacles and buffer zones. Although this strategy ensures that sensors are not in the constraint area through pre-screening, it completely relies on the mechanical distribution of regular grids and does not optimize the layout in combination with the geometric characteristics and positioning requirements of the monitoring area, resulting in overly regular distribution in wide-areas (such as the left and right sides), which cannot adjust adaptively to optimize positioning accuracy; At the edges of the monitoring area (such as the left bottom) and the transition area near the obstacles, there are some coverage blind areas, and the GDOP distribution stability is not so good.

4.2.2 Comparison of Positioning Accuracy of GDOP Thermal Distribution

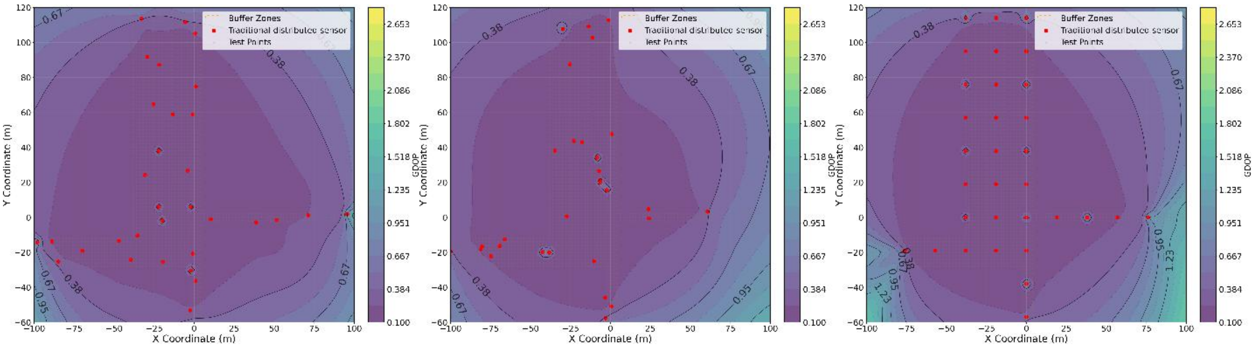

Fig. 6 GDOP of (a) FDHCI optimized array (b) Basic NSGA-II optimized array (c) Traditional array

From the GDOP thermal map above (the darker the color, the smaller the GDOP and the higher the positioning accuracy), it can be seen that:

FDHCI optimized array: The purple area (with GDOP < 0.4) occupies the largest proportion and has the most uniform color distribution, with no obvious high GDOP hotspots. This indicates that its positioning accuracy is more stable and superior throughout the entire monitoring area.

Basic NSGA-II optimized array: The purple area is slightly smaller, and there is a color fluctuation (such as the right edge being slightly blue) indicating that the stability of positioning accuracy is slightly inferior to the FDHCI scheme.

Traditional uniform array: The purple area is the smallest, and there is a yellowish high GDOP area at the edge (such as the right side), indicating the poorest positioning accuracy and coverage range. Especially in the edge areas, the accuracy tends to decrease - this defect stems from its mechanically rigid grid layout, which cannot dynamically adjust the sensor positions according to the positioning requirements.

Table 2. Three array positioning accuracy parameters comparing

Parameter	FDHCI optimized array	Basic NSGA-II optimized array	Traditional array
Average constraint violation	0	0.7147	0
Area with GDOP<0.4	74.3%	70.1%	69.9%

The quantitative indicators in Table 2 further validate the performance advantages of the FDHCI strategy:

For average constraint violation degree: The average constraint violation degree of the FDHCI optimized array is 0, achieving complete compliance; while the average constraint violation degree of the basic NSGA-II optimized array is 0.7147, indicating a significant risk of constraint violation; although a constraint violation degree of 0 through pre-screening is achieved by the traditional array, there even exists insufficient performance which is caused by mechanical layout limitations.

High positioning accuracy area proportion (area of GDOP < 0.4): The proportion of the FDHCI optimized array is 74.3%, significantly higher than the 70.1% of the basic NSGA-II and the 69.9% of the traditional array, indicating that a larger range of high-precision positioning capabilities throughout the monitoring area is provided.

5. Summary

A two-objective optimization model with the minimization of average GDOP (to enhance positioning accuracy) and the lowest constraint violation degree (to ensure compliance) as the optimization goals is constructed to address the problem of sensor layout limitations in wideband acoustic monitoring and improve the positioning accuracy and engineering applicability of the TDOA positioning system,. It proposes the FDHCI feasible region hard constraint initialization strategy, which ensures that the initial population completely falls within the feasible region by rejecting sampling. Combined with the non-dominated sorting and constraint dominance principles of the NSGA-II algorithm, it realizes evolutionary optimization. Meanwhile, three types of 32-element array schemes (FDHCI optimized array, basic NSGA-II optimized array, and traditional optimized uniform array) are designed for comparison. Experimental results show that the average constraint violation degree of the FDHCI optimized array is 0, the basic NSGA-II is 0.7147, and the traditional array achieves compliance through preset areas but has a rigid layout. In terms of positioning accuracy, the high-precision area (GDOP < 0.4) of the FDHCI optimized array accounts for 74.3%, significantly higher than 70.1% of the basic NSGA-II and 69.9% of the traditional array. In terms of convergence efficiency, the constraint violation degree of the FDHCI strategy converges to 10^{-7} within 40 generations, which is 2.5 times faster than the stagnant 10^{1} level of the basic NSGA-II. In summary, the FDHCI strategy effectively solves the problem of slow convergence and easy local optimization of traditional NSGA-II in constraint optimization. The comparison of the three array schemes confirms that it performs optimally in terms of constraint compliance, positioning accuracy stability, and convergence efficiency. It provides a systematic solution for acoustic array design in complex constraint environments. In the future, it will be expanded to combine actual acoustic propagation models (such as reflection and attenuation effects) to further improve the algorithm's engineering adaptability, and explore dynamic optimization of the number of sensors to balance cost and performance.

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