

Study on Static Load Sharing Characteristics of Coaxial Reversal Herringbone Gear Transmission System Considering Error

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Abstract. In the current high-speed helicopter main reducer, coaxial reversal herringbone gear transmission system (CRHGTS) has the advantages of light weight, high stability and long service life. In order to study the static load sharing characteristics of coaxial reversal herringbone gear transmission system, considering the factors such as transmission chain stiffness, installation error and manufacturing error, the torque balance equation of the system is established, the deformation coordination condition of the system is deduced, and the influence of manufacturing and installation errors on the load sharing performance of the system under single and combined action of gears is analyzed. The result shows that when the gear has manufacturing error, the load sharing coefficient fluctuates periodically. When the manufacturing error acts on the reversal gear and the output gear at the same time, the load sharing performance will be reduced. When the gear has installation error, the load sharing coefficient of the system is linear. The increase of gear manufacturing error and installation error will lead to the increase of load sharing coefficient and reduce the load sharing performance of the system.

Keywords: Coaxial reversal; Herringbone gear; Error; Static; Load sharing.

1. Introduction

In the configuration of the helicopter main reducer, the herringbone gear transmission system has high efficiency and stable transmission. However, due to the manufacturing and installation errors of the parts, the uniform distribution of the load transfer process of each branch cannot be fully realized, which affects the load sharing characteristics of the system, thus affecting the reliability of the whole machine. Therefore, the analysis of load sharing characteristics is one of the important work contents in the design of gear split torque transmission.

Liu [1] established a six-branch herringbone gear model, and obtained stiffness and error values with higher load sharing coefficients; Zhang [2] established a power two-branch gearing system by considering the installation error and manufacturing error, and obtained the functional relationship between the error and the equal load coefficient; Han [3] investigated the effect of the mesh phase difference between the gear pairs of a straight-tooth cylindrical gear splitting torsion system on the static equal load characteristics of the system; Lin [4] solved and analyzed the herringbone gears by using the Gaussian elimination method, etc. by considering the factors such as tooth flank clearance, error and so on. Dynamic mean load characteristics; Che [5] used 3D-LTCA method to establish TVMS model to analyses the effect of eccentricity error on the mean load coefficient, and found that the mean load characteristics of the system can be optimized by adjusting the eccentricity error; Dong [6] established a coaxial-face gear power shunt transmission system by considering the factors of error, stiffness, and floating, and found that the static mean load coefficient would show an irregular periodic fluctuations; Li [7] established a face gear power shunt system to study the effects of dry gear support stiffness, torsional stiffness, etc. on the mean load characteristics;

In this paper, the coaxial reversal herringbone gear transmission system is taken as the research object, and the static analysis model considering the factors such as transmission chain stiffness, installation error, manufacturing error and meshing phase difference is established. The torque balance equation and deformation coordination condition are derived, and the influence of the manufacturing error and installation error of each branch on the load sharing characteristics of the system is analyzed, which provides a reference for the structural design and performance analysis and optimization of the transmission system.

2. Static modelling of CRHGTS

As shown in Fig.1 is CRHGTS model, the system in the I-stage gearing including I-stage input helical cylindrical gear Z1, helical cylindrical gear Z2; II-stage gearing including II-stage small bevel gear Z3 and Z4, large bevel gear Z5 and Z6; III-stage gearing including III-stage pinion herringbone gear Z7 and Z8, III-stage upper and lower herringbone gears Z10 and Z9.

When CRHGTS is statically analyzed, it is assumed that the support and gear tooth meshing are equivalent to a spring connection, and the wheel body is equivalent to a rigid body; the torsion splitting shaft and the duplex shaft are flexible shafts; and the effects of friction, thermal expansion force and inertia force are ignored.

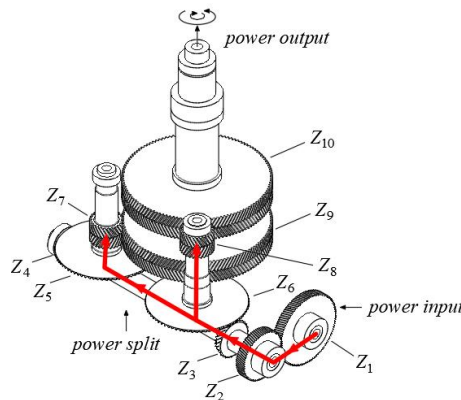


Fig.1 Model of CRHGTS

2.1 Equation of balance of moments of the system

Set the driving torque is positive, the load torque is negative, the establishment of the system's hydrostatic moment equilibrium conditions as shown in the following equation: so can be simplified to get containing T_{1_2} , T_{3_6} , T_{4_5} , T_{7_9} , $T_{8_{10}}$ of the five unknown torque of the four independent equations composed of the set of equations, to get the driveline system torque equilibrium conditions, such as the formula (1):

$$\begin{cases} T_{in} - T_{1_2} = 0; & -T_{1_2} \cdot \frac{r_{b2}}{r_{b1}} + T_{3_6} + T_{4_5} = 0; \\ -T_{3_6} \cdot \frac{r_{b6}}{r_{b3}} + T_{8_{10}} = 0; & -T_{4_5} \cdot \frac{r_{b5}}{r_{b4}} + T_{7_9} = 0; \end{cases} \quad (1)$$

Where, T_{in} is the input torque of the system; T_{1_2} , T_{3_6} , T_{4_5} , T_{7_9} , $T_{8_{10}}$ are the torque received by each gear; r_{bi} ($i=1, 2, \dots, 10$) is the radius of the gear indexing circle.

$$\begin{cases} \Sigma \left[\frac{T_{i-j}}{r_{bi} \cos \alpha_n} \cdot \cos X_{i-j} \right] - K_{xi} x_i = 0 \\ \Sigma \left[\frac{T_{i-j}}{r_{bi} \cos \alpha_n} \cdot \sin X_{i-j} \right] - K_{yi} y_i = 0 \end{cases} \quad (2)$$

Considering the role of elastic support, to establish the system considering the elastic

deformation of the bearing hydrostatic moment balance equation, such as equation (2).

Where , K_{xi} and K_{yi} are the support stiffness in x and y direction, X_{i_j} is the positive included angle between the meshing line of each gear and the x -axis.

2.2 Deformation coordination conditions

In the meshing action of gear i with gear j , gear i produces gear torsional angular displacements of γ_i and γ_j relative to gear j under the action of torque T_{i_j} , and the relative torsional angular displacements of each gear pair meshing γ_{i_j} .

$$\gamma_{i_j}(T_{i_j}) = \gamma_i - G_{i_j} \gamma_j \tag{3}$$

The relationship between the angles of the relative meshing points of the gears in the 2 branch channels can be expressed as:

$$\begin{cases} \gamma_{1_2} + G_{1_2} \gamma_{2_3} + G_{1_2} \gamma_{3_6} + G_{1_2} G_{3_6} \gamma_{6_8} + G_{1_2} G_{3_6} \gamma_{8_{10}} \\ = \gamma_1 - G_{1_2} G_{3_6} G_{8_{10}} \gamma_{10}; \\ \gamma_{1_2} + G_{1_2} \gamma_{2_3} + G_{1_2} \gamma_{3_4} + G_{1_2} \gamma_{4_5} + G_{1_2} G_{4_5} \gamma_{5_7} + G_{1_2} G_{4_5} \gamma_{7_9} \\ = \gamma_1 - G_{1_2} G_{4_5} G_{7_9} \gamma_9; \end{cases} \tag{4}$$

Where , $G_{3_6}=G_{4_5}$, $G_{7_9}=G_{8_{10}}$.

The deflection angle resulting from the torsional deformation of the split-twist and duplex shafts can be expressed by the following formula:

$$\gamma_{tsi_j}(T_{i_j}) = \frac{T_{i_j}}{K_{tsi_j}} \tag{5}$$

Where, K_{tsi_j} is the torsional stiffness of the shaft; T_{i_j} is the torque applied to the shaft.

The equivalent meshing error γ_{Mi} obtained from the projection of the manufacturing error is:

$$\gamma_{Mi} = \gamma_{mi} + \gamma_{ai} = m_i \sin(\alpha_n + \beta_{mi} + \omega_i t) + a_i \sin(\alpha_n + \beta_{ai}) \tag{6}$$

Where, γ_{mi} is the manufacturing error; γ_{ai} is the installation error; m_i and a_i are the amplitude of the manufacturing and installation errors of each gear pair, respectively; α_n is the pressure angle; β_{mi} and β_{ai} are the manufacturing and installation error azimuth angles of each gear pair, respectively; t is the time; and ω_i is the angular velocity of the main wheel.

The relative angular displacement of the axial deformation of each gear component of the system under elastic support $\gamma_{RD_{i_j}}$ is:

$$\gamma_{RD_{i_j}} = \frac{[(x_i - x_j) \cos X_{i_j} + (y_i - y_j) \sin X_{i_j}]}{r_{bi}} \tag{7}$$

Where, x_i , x_j , and y_i , y_j represent the deformation in the x and y directions respectively. The relative angular displacement of the axial deformation and the comprehensive meshing error are superimposed to obtain the comprehensive angular displacement γ_{Ci_j} of each gear pair as follows:

$$\gamma_{Ci_j} = \gamma_{RD_{i_j}} + \gamma_{Mi} \tag{8}$$

Considering the torsional angle deformation caused by meshing force, the meshing transmission error of the gear pair can be obtained:

$$\gamma_{i_j} = \frac{T_{i_j}}{K_{i_j} r_i r_{bi} \cos \alpha_n} + \gamma_{Ci_j} \tag{9}$$

3. Analysis of the influence of comprehensive meshing error on load sharing coefficient

3.1 Load sharing characteristics calculation

In this paper, the input power CRHGTS is $P = 2800\text{Kw}$, and input speed is $n = 9000\text{r/min}$. The above established load-sharing analysis model is solved by the Least Square Method. The gear and stiffness parameters of the CRHGTS are shown in Table 1, Table 2 and Table 3, respectively.

Table 1. Gear parameters of the CRHGTS

Parameter	Tooth number	Modules(mm)	Pressure angle(°)	Helix angle(°)
Gear Z_1	29	3	20	11
Gear Z_2	73	3	20	11
Gear Z_3 and Z_4	31	5.5	20	15
Gear Z_5 and Z_6	71	5.5	20	15
Gear Z_7 and Z_8	22	5.5	20	30
Gear Z_9 and Z_{10}	122	5.5	20	30

Table 2. Stiffness parameters of the CRHGTS

Parameter	Value/(N·m ⁻¹)
K_{1_2}	1.95×10^9
K_{3_6}, K_{4_5}	1.85×10^9
K_{7_9}, K_{8_10}	2.35×10^9
K_{ts2_3}	2.04×10^7
K_{ts3_4}	1.15×10^7
K_{ts5_6}	3.74×10^5
K_{ts6_8}	3.24×10^5

Table 3. Stiffness parameters of the CRHGTS

Parameter	Value/(N·m ⁻¹)
K_{x1}, K_{y1}	1.45×10^9
K_{x2}, K_{y2}	1.48×10^9
$K_{x3}, K_{y3}, K_{x4}, K_{y4}$	1.34×10^9
$K_{x5}, K_{y5}, K_{x6}, K_{y6}$	5.14×10^9
$K_{x7}, K_{y7}, K_{x8}, K_{y8}$	1.98×10^9
$K_{x9}, K_{y9}, K_{x10}, K_{y10}$	7.01×10^9

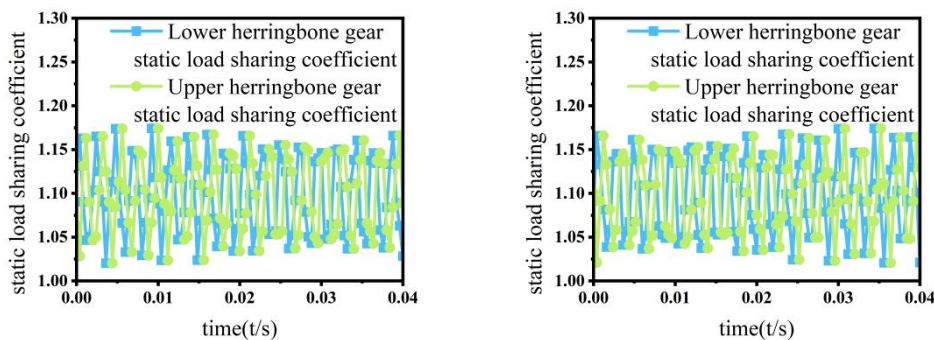
According to the moment equilibrium equations of the transmission system, the elastic deformation coordination conditions and the static equilibrium conditions of the elastic support, the solution calculation can be obtained by the system transmission torque $T_{i,j}$, defining the system static load sharing coefficient as shown in the following equation.

$$\varphi_i = \max\left[\frac{T_i}{T}\right] \tag{10}$$

Where, T_i is the actual torque; T is the theoretical torque.

3.2 Influence of manufacturing error on the load sharing characteristics

When the Z_3 is combined with the Z_7 and the gear Z_8 at a manufacturing error of $5\mu\text{m}$, the load sharing coefficient curve is obtained as shown in Fig.2.



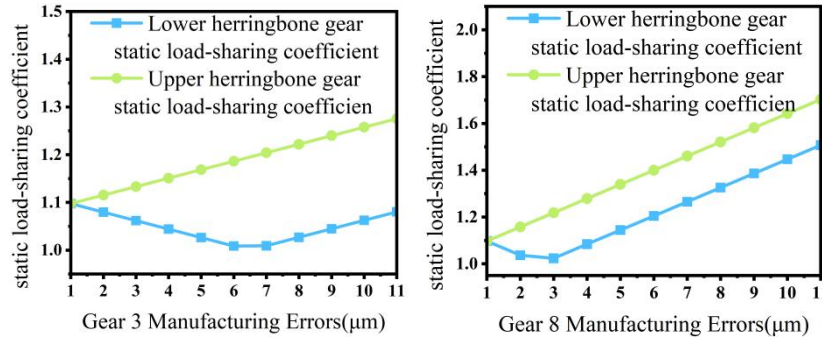
(a) Z_3 and Z_7 manufacturing errors combined (b) Z_3 and Z_8 manufacturing errors combined

Fig.2 Multi-gear manufacturing error influence curve

When the manufacturing errors of different side gears Z_3 and Z_7 work together, the load sharing coefficient of the upper herringbone gear and lower herringbone gear changes between 1.02 and

1.175. When the manufacturing errors of the same side gears Z_3 and Z_8 are combined, the load sharing coefficient of the upper herringbone gear and lower herringbone gear changes between 1.02 and 1.18. The results show that the load sharing performance is worse under the joint action of the manufacturing errors of the same side gear.

When the manufacturing errors of gears Z_3 and Z_8 are increased from $0\mu\text{m}$ to $50\mu\text{m}$ respectively, the load sharing coefficient curve is shown in Fig.3.

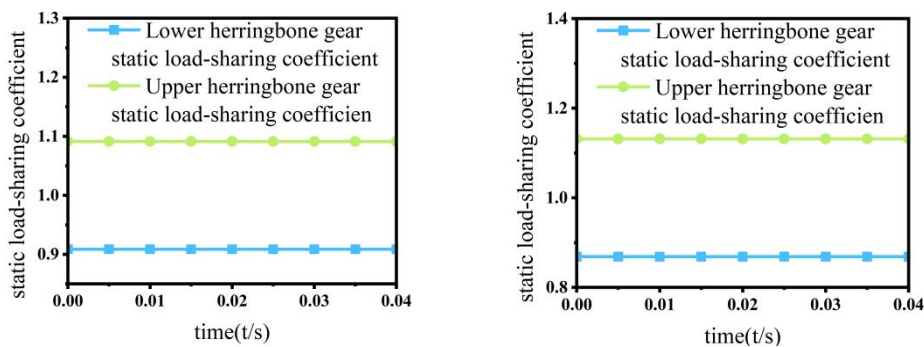


(a) Change in manufacturing error of Z_3 (b) Change in manufacturing error of Z_8
 Fig.3 Influence curve of single gear manufacturing error variation

In Fig.3(a), with the increase of the manufacturing error of Z_3 , the load sharing coefficient of the herringbone gear on the output stage shows an upward trend, and the load sharing coefficient of the herringbone gear under the output stage shows a trend of decreasing first and then increasing, and the load sharing coefficient increases by 14.1%. The reason is that when the manufacturing error is small, the uneven load caused by the stiffness of the system is compensated. When the manufacturing error increases from $25\mu\text{m}$ to $50\mu\text{m}$, the load sharing coefficient of the system shows an upward trend. In Fig.3(b), with the increase of the manufacturing error of Z_8 , the load sharing coefficient of the upper herringbone gear increases by 35.3%, and the lower herringbone gear increases by 25%.

3.3 Influence of installation error on the load sharing characteristics

When the gear Z_3 is combined with the gear Z_7 and the gear Z_8 at the installation error of $5\mu\text{m}$, the load sharing coefficient curve is obtained as shown in Fig.4. When the installation errors of different side gears Z_3 and Z_7 work together, the load sharing coefficients of the upper and lower herringbone gears are 1.09 and 0.91, respectively. When the installation errors of the same side gears Z_3 and 8 are combined, the load sharing coefficients of the upper and lower herringbone gears are 1.13 and 0.87, respectively. The results show that the load sharing performance is worse under the combined action of the installation errors of the same side gears.



(a) Z_3 and Z_7 installation errors combined (b) Z_3 and Z_8 installation errors combined
 Fig.4 Multi-gear installation error influence curve

When the installation error of gear Z_3 and Z_7 increases from $0\mu\text{m}$ to $50\mu\text{m}$, the load sharing coefficient curve is shown in Fig.5. In Fig.5(a), as the installation error of gear Z_3 increases, the load sharing coefficient of output stage gear decreases first and then increases. In Fig.5(b), as the

installation error of gear Z_8 increases, the load sharing coefficient of output stage gear increases, and the load sharing coefficient increases by 8.3%.

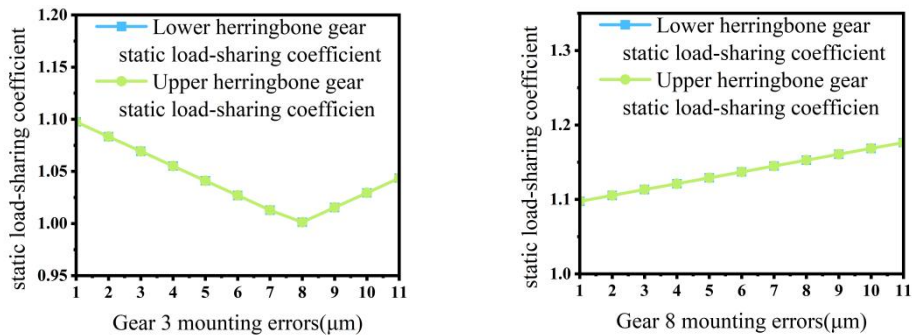
(a) Change in installation error of Z_3 (b) Change in installation error of Z_8

Fig.5 Influence curve of single gear mounting error variation

4. Summary

In this paper, by comprehensively considering factors such as meshing stiffness, torsional stiffness, support stiffness, elastic deformation, manufacturing error, installation error, etc., a static model of CRHGTS is established, and the static load sharing characteristics of the system under the influence of gear manufacturing error and installation error alone and jointly are analyzed.

(1) When the manufacturing error of a single gear changes, the static load sharing coefficient fluctuates sinusoidally with time. When the gear installation error changes, the static load sharing coefficient does not fluctuate with time.

(2) Increasing the manufacturing error and installation error will lead to a larger load sharing coefficient of the system. The manufacturing error has a greater impact on the characteristics of the system. The manufacturing error of the gear Z_8 increases from $0\mu\text{m}$ to $50\mu\text{m}$, and the load sharing coefficient increases by 35.3%.

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References

- [1] Liu Linlin, Wang Sanmin, Zou Haoran, et al. Research on load-sharing characteristics of six-branch herringbone gear transmission system. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2023, 237(4): 655-665.
- [2] Zhang Ting, Li Yuxi, Wang Sanmin. Static load sharing characteristics of power two branch gear transmission system. Mechanical transmission, 2012, 36 (03): 14-16 + 24.
- [3] Han Ning, Zhu Rupeng, Chen Weifang. Effect of meshing phase difference on static load sharing characteristics of co-axial dual-output cylindrical gear split-torque-parallel transmission. Journal of Mechanical Transmission, 2025, 49 (01): 1-10.
- [4] Lin He, Wang Sanmin. Effects of parameters on dynamic load and load sharing characteristics of split torque-parallel gear transmission system. Journal of Mechanical Transmission, 2019, 43 (05): 18-22 + 63.
- [5] Che Xiaoyu, Zhang Chao, Yu Hu, et al. The effect of eccentricity error on the static load sharing characteristics of PTBTS based on 3D-LTCA method. Proceedings of the Institution of Mechanical Engineers, 2025, 239(8): 3117-3130.
- [6] Dong Hao, Zhang Haoqin, Liang Gang, et al. Influence of errors and floating on static load-sharing characteristics of twin rotors concentric face-gear power-split transmission system. Journal of Mechanical Science and Technology, 2023, 37 (4): 1883-1901.

- [7] Wang Li, Gao Yufeng, Zhang Jin. Study on the dynamic load sharing characteristics of nutation face gear transmission. Discover Applied Sciences, 2025, 7(2): 111-111.