Accelerate the AHP Method by Indicator SubGroup

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Abstract. The Analytic Hierarchy Process (AHP) method is an effective and commonly used approach for determining indicator weights in an index system. It establishes weights for each indicator through pairwise comparisons and specific algorithms. However, AHP faces three main challenges: (1) Difficulty in precisely quantifying pairwise relative ratios between indicators; (2) Impracticality when handling excessive indicators due to the overwhelming number of required pairwise comparisons; (3) Scarcity of experts sufficiently familiar with all indicators in large-scale systems. To address these issues, this paper proposes the AHP with Indicator Subgroup method. This innovative method categorizes indicators into multiple subgroups based on importance levels, then separately applies AHP to determine: (1) Weights among indicator subgroups, and (2) Weights of individual indicators within each subgroup. With these two weights, the final weights of each indicator in the index system is calculated and obtained. This approach effectively resolves AHP's limitations and enhances its feasibility. This paper also demonstrates the method's effectiveness through application to a residential environment health index system.

Keywords: AHP, Index system, residential environment, health.

1. Introduction

The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions by deriving indicator weights through pairwise comparisons. It's a classical multi-criteria decision-making method, and has achieved remarkable breakthroughs in both theoretical innovation and practical applications in recent years. Theoretically, researchers have focused on three main directions: firstly, optimizing the consistency check of judgment matrices through fuzzy AHP (FAHP)[1], interval AHP[2], and machine learning-based methods (e.g., genetic algorithms, neural networks)[3]; secondly, deeply integrating AHP with other decision-making methods, such as combining with TOPSIS for alternative ranking or integrating with DEMATEL to analyze causal relationships in complex systems; thirdly, developing dynamic AHP models by introducing time-series weight indicators and real-time big data updating mechanisms to enhance decision-making timeliness. In terms of applications, innovative uses of AHP are mainly reflected in emerging fields such as sustainable energy management (e.g., GIS-based site selection assessment for wind/solar farms)[4], healthcare (optimizing vaccine distribution strategies during pandemics)[5], smart manufacturing (supplier selection and automation solution evaluation)[6], fintech (cryptocurrency investment decision analysis)[7], and work safety[8]. Current research challenges primarily include controlling subjective judgment biases, optimizing algorithms for high-dimensional data processing, and improving adaptability to dynamic environments. Future development trends will focus on deep integration of AHP with artificial intelligence, the development of real-time decision support systems, and further expansion of interdisciplinary applications. In practical applications, the AHP method has problems such as difficulty in accurately determining the relative importance of indicators, being hard to control the excessive number of indicator comparisons, and experts' difficulty in being familiar with all indicators. This paper will propose solutions to address these issues.

Here is an introduction to the main content of this paper. In Section 2, we will conduct a detailed analysis of the main problems existing in the application of the AHP method and propose our original solutions. In Section 3, we provide a formal description of the original algorithm. Section 4 introduces an index system we established for evaluating the health of residential environments. In Section 5, we apply the algorithm proposed in this paper to optimally determine the weights of each indicator in the index system for evaluating the health of residential environments, demonstrating the efficiency advantages of the algorithm proposed in this paper. Finally, we present the analysis and summary of the full text.

2. Basic Idea

2.1 AHP Method's Problems

Although the AHP method is an effective and commonly used method for determining the weights of indicator systems,there are still many challenges in efficiently using this method. One of the difficulties in applying this method is that it is difficult to determine the comparative relationship between pairwise indicators relatively accurately. Even if the relative ratio is limited to integers from 1 to 9 according to general practice, it is still very difficult to achieve. Experts generally find it easier to distinguish between 3 to 5 levels. The second difficulty is that it is difficult to ensure the rationality of the comparison relationship between indicators. If the consistency of the comparison matrix does not meet the requirements, it is necessary to adjust the comparison matrix. There are also 45 indicators that need to be adjusted in a 10 indicators system, which is very significant in terms of operability. The third difficulty is that when there are relatively many indicators, it is difficult to find experts who are very familiar with all of the indicators. Generally speaking, it is difficult for one expert to make objective and scientific judgments on the relative importance of all indicators.

2.2 AHP with Indicator Subgroup Method

This article proposes an original method to solve the above-mentioned problems of the Analytic Hierarchy Process (AHP) by establishing indicator group to improve its efficiency. For example, in an indicator system with ten indicators, they are first divided into three hierarchical levels of indicator subgroups, each containing multiple indicators of similar importance. Using AHP, the average relative weight of indicators at different levels is determined, establishing the average weight for each indicator within its group. Then, within each group, AHP is applied again to determine the relative weight ratio of each indicator, ultimately yielding the weight ratio for every indicator in the entire system. Given the similar importance of indicators within the same group, the relative importance ratio used during intra-group AHP comparisons can be restricted to a small set of values, specifically three values greater or equal to 1. Conversely, when applying AHP between different groups, the relative importance ratio can utilize four distinct values. Furthermore, these assigned values follow an exponential rather than linear distribution; for instance, the three intra-group importance ratios can be [1, 1.4, 2], while the four inter-group importance ratios can be [1, 2, 4, 8]. Below, we formally describe the AHP with Indicator Subgroups Method proposed in this article.

3. Formal Description of the Algorithm

3.1 Abbreviations and Acronyms

The AHP with Indicator Subgroup Method organizes the original n indicators a_j into m subgroups, where the i-th subgroup SG_i contains k_i indicators, in some locations, we use ki in place of k_i. The inter-group weight vector wg, derived via AHP, represents relative weights between subgroups, the components of wg are represented by $w_{g,i}$. While intra-group weight vectors w_{g_i} (also AHP-derived) assign relative weights to indicators within each SG_i, the components of w_{g_i} are represented by $w_{g\_i,h}$. For any indicator $a_{i,h}$ (denoting the h-th indicator in subgroup i), its final system weight is denoted as $w_{i,h}$.

3.2 The Algorithm

Algorithm: AHP with Indicator Subgroup
$S_{indicator}=\{a_1,a_2,...,a_n\}$
1: Create empty sets $SG_i(i\in\{1,2,...,m\})$
2: Assign a_j to SG_i ,one a_j only belongs to one SG_i and $SG_i=\{a_{i\_1},a_{i\_2},...,a_{i\_ki}\}$
3: w_g= AHP($g_1,g_2,...,g_m$)
4: w_{g_i}= AHP($a_{gi\_1},a_{gi\_2},...,a_{gi\_ki}$)
5: $w_{i,h}= k_i*w_{g\_i,h}*w_{g,i}/(\sum_{i=1}^{m}\sum_{h=1}^{ki} k_i*w_{g,i}*w_{g\_i,h})$

Fig. 1 AHP with Indicator Subgroup Method

As shown in figure 1, the AHP with Indicator Subgroups method comprises five sequential steps:

(1) Establish hierarchical indicator subgroups;

(2) Distribute indicators with similar weights into each subgroup;

(3) Compute inter-group relative weight vectors wg using AHP under the average weighting scheme;

(4) Calculate intra-group relative weight vectors wg_i for each subgroup using AHP;

(5) Determine each indicator's final weight through the formula (1):

$$w_{i,h}= k_i*w_{g\_i,h}*w_{g,i}/(\sum_{i=1}^{m}\sum_{h=1}^{ki} k_i*w_{g,i}*w_{g\_i,h}) \quad (1)$$

4. Residential Environmental Health Index System

4.1 Residential Environmental Health Background

Residential Environmental Health refers to the quality of living conditions in residential areas and their direct impact on human physical and mental well-being. A objective and scientific indicator system is very necessary. Because we want to ranslates complex environmental data (e.g., PM2.5 levels, heavy metal contamination) into actionable metrics， comprehensively consider the impact of physical, biological, and socio-ecological indicators on human health, then Identifies priority areas for improvement, and directs funding/infrastructure projects to zones with the lowest environmental health scores. When building The health index system of the residential environment, we referred to “The WELL Building Standard™ version 2” (WELL v2™) and “The Healthy Building Evaluation Standard". "WELL v2™” is a building standard, it is hoped to establish a well - considered building health evaluation index system to promote the health of residents. “WELL v2™” contains ten indicators including air, water, nourishment, light, movement, thermal, sound, materials, mind, community. "The Healthy Building Evaluation Standard" is a group standard formulated by the China Architecture Association. Following the principle of multidisciplinary integration, the standard comprehensively evaluates indicators such as building air, water, comfort, fitness, human culture, and services.

4.2 The proposed Residential Environmental Health Index

The health assessment index system for residential environments discussed in this paper examines the impact of the environment on human health from a community perspective. Therefore, the index system does not include some city-level indicators such as meteorology and food safety. Nor does it target indicators that each household can adjust to a large extent, such as thermal environment and ergonomics within living units.

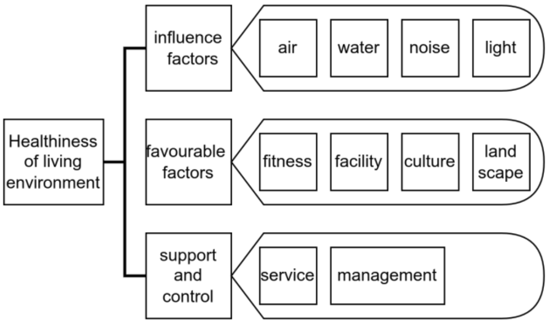

Fig. 2 Proposed Residential Environmental Health Index

The health index system of the residential environment established in this paper includes three categories of indicators. The first category is indicators that provide basic support for human health, safety, and comfort. The second category is indicators that enhance human psychological and physical health. The third category is indicators that characterize the community's ability to manage and improve the first and second category indicators through management means. The first category of indicators includes air, water, noise, light. The second category of indicators includes fitness and facilities, community cultural environment and community landscape, etc. The third category includes community-provided services and the management level of health-related indicators. Since this article has limited space, the definition of indicators will not be discussed in detail.

5. Apply AHP with Indicator Subgroup Method

5.1 Residential Environmental Health Background

The following section presents the detailed procedure for applying the AHP with Indicator Subgroups Method, proposed in this study, to determine indicator weights within the residential environmental health index system.

Through organizing expert discussions, the 10 indicators were divided into three groups based on their importance. The firse group includes four indicators: air, water, civility, and management.The second group includes three indicators: light, noise and service.The third group includes three indicators: fitness, facility, and landscape.

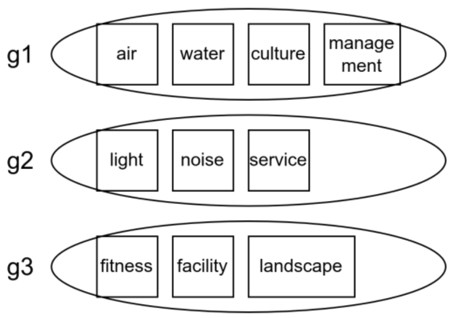

Fig. 3 Proposed Residential Environmental Health Index

To make the following research more concise, we denote these 10 indicators as f1 to f10, as detailed in Table 2.

Table 2. Descriptors for indicators

Indicator list1	descriptor	Indicator list2	descriptor
air	f1	facility	f6
water	f2	culture	f7
light	f3	landscape	f8
noise	f4	service	f9
fitness	f5	management	f10

Figure 4 presents both the intergroup pairwise comparison matrix (between groups one, two, and three) and the intragroup pairwise comparison matrices (within groups one, two, and three).

$$\begin{pmatrix} & g1 & g2 & g3 \\ g1 & 1 & 2 & 4 \\ g2 & 1/2 & 1 & 2 \\ g3 & 1/4 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} & f1 & f2 & f7 & f10 \\ f1 & 1 & 1.4 & 2 & 2 \\ f2 & 1/1.4 & 1 & 1.4 & 1.4 \\ f7 & 1/2 & 1/1.4 & 1 & 1 \\ f10 & 1/2 & 1/1.4 & 1 & 1 \end{pmatrix} \begin{pmatrix} & f3 & f4 & f9 \\ f3 & 1 & 1 & 2 \\ f4 & 1 & 1 & 2 \\ f9 & 1/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} & f5 & f6 & f8 \\ f5 & 1 & 1 & 1.4 \\ f6 & 1 & 1 & 1.4 \\ f8 & 1/1.4 & 1/1.4 & 1 \end{pmatrix}$$

Inter Groups Intra Group One Intra Group Two Intra Group Three

Fig. 4 Pairwise comparison matrix for intra and inter groups

5.2 Gets the Final indicators's weights

After establishing indicator groupings and obtaining pairwise comparison matrices both between and within the groups, the Analytic Hierarchy Process (AHP) method is used to determine the weights for inter-group and intra-group pairwise comparisons. The weight of each indicator within the residential environment health index system is then calculated using Equation (1). Due to space constraints, the detailed calculation process is not provided. The final computed weights are presented in the Table 3.

Table 3. Final indicators's weights

Indicator list1	weight(%)	Indicator list2	weight(%)
air	23.6	facility	4.4
water	16.7	culture	11.9
light	8.8	landscape	3.2
noise	8.8	service	6.3
fitness	4.4	management	11.9

The proposed AHP with Indicator Subgroup Method significantly reduces the number of pairwise judgments experts need to make compared to traditional AHP. As shown above, a 10-indicator system, our approach requires only 15 judgments versus the traditional 45. For a 15-indicator system, traditional AHP needs 105 judgments (15×14/2), while our method requires just 33 (3 for group weighting + 3*10 for intra-group comparisons). Crucially, experts can focus only on subsets of indicators they understand.They assess only within their specialized group (e.g., just around 10 judgments for a small subset).When evaluating groups, they make cross-group comparisons (e.g., around 3 judgments).

This delivers three key advantages:

(1) Eliminates expertise barriers: Experts no longer need familiarity with all indicators.

(2) Reduces expert burden: Cuts workload by >60% (e.g., 33 vs. 105 judgments).

(3) Maintains consistency: Avoids significant comparison inconsistencies despite partial participation. Ultimately, this makes expert recruitment easier and accelerates the AHP process.

6. Summary

In this paper, we analyze the main drawbacks of the AHP method and innovatively propose an enhanced approach: the AHP with Indicator Group Method. Based on this framework, we construct a residential environmental health index system comprising ten key indicators. By applying the proposed method, we efficiently and systematically derive the weights of these indicators. Case studies are presented to demonstrate the effectiveness and practicality of this approach.

The proposed method offers the following advantages:

(1) **Reduced comparison workload:** It significantly decreases the number of required pairwise comparisons, maintaining feasibility even for indicator systems with up to more than 50 parameters (e.g., eight groups with eight indictors in one group).

(2) **Flexible prioritization:** It effectively emphasizes critical indicators while retaining adaptability, when indicators have similar importance, the number of sub-levels can be accordingly reduced.

(3) **Modular design:** The method enables problem decomposition, allowing distributed work among multiple experts while minimizing inconsistencies in evaluation criteria.

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