

An Efficiency Evaluation Method

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Abstract. This article proposes a method for evaluating damage effectiveness, which includes methods for calculating missile target intersection, calculating damage effectiveness, analyzing damage effectiveness, and characterizing damage effectiveness. The calculation method for missile target intersection includes coordinate system transformation method and end trajectory calculation method. The calculation methods for damage effectiveness include component damage effect calculation, component damage effectiveness calculation, and target damage effectiveness calculation. Combined with practical use, good results have been achieved.

Keywords: effectiveness evaluation, destructive effect, missile target rendezvous.

1. Introduction

Efficiency evaluation is a widely researched field in the industry. This article proposes a method for evaluating destructive efficiency, which includes methods for calculating missile target intersection, calculating destructive efficiency^[1], analyzing destructive efficiency, and characterizing destructive efficiency.

2. Calculation method for missile target intersection

The intersection of projectile and target is achieved through given intersection conditions, including the basic parameters of the warhead (end velocity attitude, aiming point, fuse parameters) and target parameters (target type, position). Shooting line tracking technology is used to calculate the intersection between the warhead and its destructive elements and various targets. After calculation, information such as the position of the destructive element hitting the target and the list of impact components is provided.

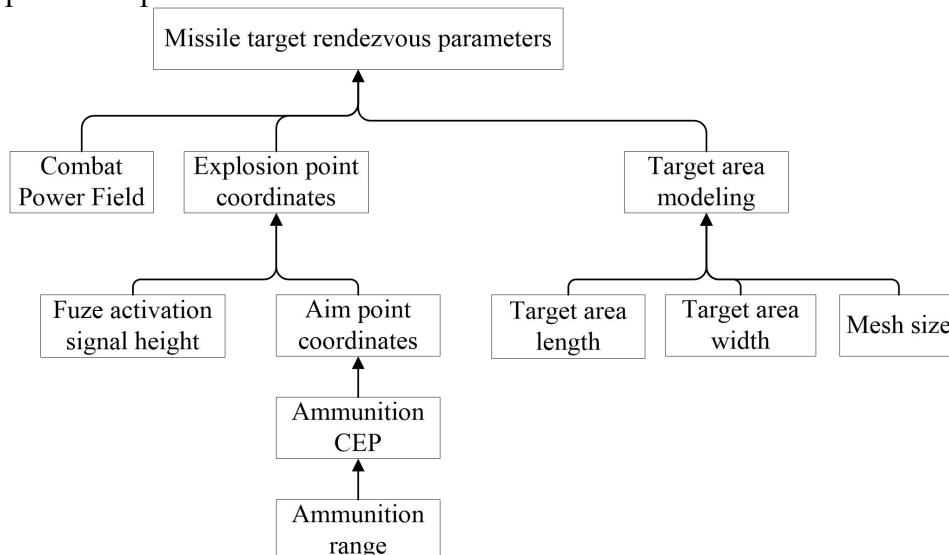


Fig.1 Calculation method for missile target intersection.

The calculation of the intersection of projectile and target includes: coordinate transformation between the projectile coordinate system and the target coordinate system in the global coordinate system, calculation of the end trajectory, calculation of the ammunition impact point, and calculation of the ammunition explosion point^[2]. The calculation process is shown in the above figure1. Below, we will explain the steps of the calculation process.

2.1 Coordinate System Conversion Method

Here, it is necessary to first define a target coordinate system to represent the linear equation of the ammunition's end trajectory, as well as its impact point coordinates and ammunition explosion point coordinates on the target equivalent model.

We do not focus on the firing and flight process of ammunition here, but only take the end flight attitude information of ammunition as the research object to obtain the end trajectory straight line of ammunition. Obviously, the end trajectory straight line is the tangent of the ammunition's flight trajectory curve at the end. Here, the direction vector of the end trajectory line is determined by the pitch and yaw angles of the ammunition end. The pitch and yaw angles of ammunition are defined in the target coordinate system. Translating the axis of the projectile to the origin of the target coordinate system, the angle between the projectile axis and the zt axis is the pitch angle, represented by ϕ , with a range of 0° to 180° ; The angle between the projection of the projectile axis on the xtoty plane and the xt axis is the yaw angle, represented by θ , with a range of 0° to 360° .

The direction vector $v_b (x_b, y_b, z_b)$ of the end trajectory line can be obtained by the pitch angle and yaw angle, and its value is obtained by the following equation. The direction of v_b is towards the direction of ammunition flight.

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \sin \phi \cos \theta \\ \sin \phi \sin \theta \\ \cos \phi \end{bmatrix}$$

Relying solely on v_b cannot determine the final trajectory line, so it is necessary to first define a guidance plane. The guidance plane passes through the aiming point P_a of the ammunition in the target coordinate system and is perpendicular to v_b . Specifically, the straight line passing through the aiming point with VB as the direction vector is called the ideal end trajectory straight line, which is not affected by the guidance error of the ammunition and is different from the actual end trajectory straight line in the following text. A guidance coordinate system (hereinafter referred to as GCS) can be established on the guidance plane according to the definition method of the right-handed Cartesian coordinate system^[3]. It is represented as O_g with the aiming point P_a as the origin, and as z_g with the ideal end trajectory line as the Z-axis, with the positive direction opposite to the direction of ammunition flight; According to the right-hand rule, determine the X-axis and Z-axis respectively, represented as x_g and y_g .

It is not difficult to see from the definitions of GCS, target coordinate system, pitch angle ϕ , and yaw angle θ that GCS can be obtained from the target coordinate system through a series of translation and rotation transformations. Firstly, translate the target coordinate system so that its origin coincides with the aiming point P_a to obtain a new coordinate system; Step two, rotate the new coordinate system counterclockwise by θ angle around its Z-axis to obtain an intermediate coordinate system; Step three, rotate the middle coordinate system counterclockwise by an angle of ϕ around its Y-axis to obtain GCS. Based on the above geometric relationships, a coordinate value transformation relationship between GCS and the target coordinate system can be established^[4]. Assuming P_0 is any point on the target coordinate system with coordinate values of (x_0, y_0, z_0) ; The coordinates of the aiming point P_a on the target coordinate system are (x_a, y_a, z_a) . Based on the translation relationship from the target coordinate system to the new coordinate system, the coordinate values (x', y', z') of P_0 in the new coordinate system can be calculated. According to the rotation relationship from the new coordinate system to the intermediate coordinate system, the coordinate values (x'', y'', z'') of P_0 in the intermediate coordinate system are calculated as follows:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Based on the rotation relationship from the intermediate coordinate system to GCS, the coordinate values(x''' , y''' , z''')of P0 in GCS are calculated as follows:

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

Combining the above equations yields:

$$\begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi & \sin\theta\cos\varphi & -\sin\varphi \\ -\sin\theta & \cos\theta & 0 \\ \cos\theta\sin\varphi & \sin\theta\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x_0 - x_a \\ y_0 - y_a \\ z_0 - z_a \end{bmatrix}$$

The coordinate value transformation relationship between GCS and the target coordinate system.

2.2 Calculation method for end trajectory line

The analysis process of missile target intersection is shown in the following figure. As the analysis of missile target intersection is entirely conducted within the target coordinate system, it is necessary to convert the destructive elements of the explosive bomb into the target coordinate system. According to the pitch angle ν_R , yaw angle φ_R , and explosion point position H (x, y, z) of the ammunition warhead in the target coordinate system, the method and formula in the previous section can be used to convert the coordinates Psp (x, y, z) of the fragments in the projectile coordinate system to the target coordinate system, thereby obtaining the spatial distribution CPsp (x, y, z) of the fragments^[5].

The parameters for ammunition target rendezvous are functions of variables such as fuse system parameters, ammunition circular probability error (CEP), ammunition warhead power field, target area, and ammunition drop speed, as shown in the following equation.

$$\Delta_{rocktoaim} = f(\Delta_{fuze}, \Delta_{CEP}, \Delta_{war}, \Delta_{aim}, V_R)$$

In the formula:

Δ_{fuze} —Fuze system parameters;

Δ_{CEP} —Probability error of ammunition circle;

Δ_{war} —Combat power field;

Δ_{aim} —Target area;

V_R —Ammunition drop speed.

The parameters of the fuse system mainly include the high explosive area of the ammunition warhead controlled by the fuse. Due to the existence of error terms, the fuse system cannot accurately meet the fixed detonation height, generally in the form of detonation height expectation \pm detonation height standard deviation, and the function model is listed in the above equation. Within this range, it can be approximately assumed that the explosion height follows a normal distribution under a large sample size. Therefore, the computer normal distribution sampling method is used to sample and obtain the explosion height value of the ammunition warhead.

$$\Delta_{fuze} = f(\mu_H, \sigma_H)$$

The accuracy of ammunition hit is the main performance indicator for evaluating the combat capability of ammunition. In the accuracy evaluation of weapon and ammunition systems, circular probability error (CEP) is often used as a method for accuracy evaluation. The hit accuracy of the same type of weapon ammunition varies at different ranges, and CEP is a function of range, as shown in the above equation. Due to the fact that the circular probability error is defined within the guidance plane and follows a normal distribution, Monte Carlo random simulation method is used to

sample the aiming deviation within the guidance plane. Combined with the aiming point, the actual landing point of the ammunition warhead can be calculated. Based on the actual landing point, combined with the sampling results of the explosion height and the final azimuth and elevation angles of the projectile movement, determine the coordinates H (x, y, z) of the ammunition warhead explosion point in the target coordinate system.

$$\Delta_{CEP} = f(S)$$

3. Calculation method for destructive effectiveness

Calculate the damage effect of target components based on the target intersection information file obtained from missile target intersection calculation and the target structure data table constructed from target vulnerability. Input the target function data table, calculate the damage effectiveness of the component and the target based on the damage effect calculation results, and then weight them to obtain the damage effectiveness of the component and the target. Finally, output the target damage rate. The calculation of damage effectiveness includes: component damage effect calculation, component damage effectiveness calculation, and target damage effectiveness calculation^[6].

3.1 Calculation of component damage effect

3.1.1 Calculation of fragment damage effect.

During the process of fragment scattering, each time the fragment intersects with the target structure, it is equivalent to a penetration process of the fragment into the metal target plate. It is necessary to determine whether the fragment can penetrate the target plate. If it can penetrate, the residual velocity of the fragment after penetration needs to be calculated for the next penetration calculation. Therefore, the ballistic limit calculation formula needs to be used to determine whether the fragment can penetrate the target plate, and the residual velocity calculation formula needs to be used to calculate the residual velocity after penetration.

The commonly used ballistic limit calculation formula, residual velocity calculation formula, and their input-output parameters are shown in the following table.

Table3.1 .Commonly used ballistic limit velocity calculation models.

Serial Number	Formula content	Input parameter	
		Basic parameters	Other parameters
Formula 1	$v_{50} = A \cdot \frac{D^\alpha}{m_f^\beta} \cdot T^\gamma \cdot (\sec \theta_c)$	Fragment diameter and mass Fragmented with horns Target plate thickness	Correction factor Fitting coefficient
Formula 2	$v_{50} = k_1 (\overline{TA})^{\alpha_1} (m_f)^{\beta_1} (\sec \theta_c)^{\gamma_1}$	Fragment quality Fragmented with horns Target plate thickness	Average target area of fragments Material coefficient
Formula 3	$v_{50} = a \times \left(\frac{h}{d}\right)^b \times \frac{\rho_t^c}{\rho_p^d} \times \sigma_t^e$	Fragment diameter Target plate thickness Fragment material density Target board material density Ultimate strength of target plate	Fitting coefficient
Formula 4	$v_{50} = A \times \frac{h_t^a \sigma_{bt}^b \rho_t^c}{d^d \rho_p^e}$	Fragment diameter Target plate thickness Fragment material density Target board material density	Yield limit of target plate material Fitting coefficient
Formula 5	$v_{50} = A \times \left(\frac{h}{d}\right)^{a1} \times \left(\frac{\rho_t}{\rho_p}\right)^{a2} \times \left(\frac{\sigma_{st}}{\sigma_{sp}}\right)^{a3} \times \sec^{a4} \theta$	Fragment diameter Fragmented with horns Target plate thickness	Fitting coefficient

		Fragment material density Target board material density Ultimate strength of fragments Ultimate strength of target plate	
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Table3.2. Common Remaining Speed Calculation Model.

Serial Number	Formula content	Input parameter	
		Basic parameters	Other parameters
Formula 1	Vertical penetration: $v_r = \frac{1}{1 + \frac{\rho_t \left(\frac{D_t}{D}\right) T}{\rho_p \left(\frac{D}{D}\right) L}} (v_0^2 - v_{50}^2)^{1/2}$ Oblique penetration: $v_r = \frac{\cos \beta}{1 + \frac{\rho_t \left(\frac{D_t}{D}\right) T}{\rho_p \left(\frac{D}{D}\right) L \cos \theta}} (v_0^2 - v_{50}^2)^{1/2}$	Fragment diameter and length Fragment density Fragmentation angle and velocity Target plate thickness and density Plug diameter Ballistic maximum speed	Oblique penetration: Deviation angle from the target plate
Formula 2	$v_r = \sqrt{\frac{v_0^2 - v_c^2}{1 + \frac{k\pi d^2 b \rho_t}{4m \cos \alpha}}}$	Fragment diameter and mass Fragmentation angle and velocity Target plate thickness and density Ballistic maximum speed	Fitting coefficient

3.1.2 Calculation of shock wave damage effect.

The shock waves generated by explosions at different locations will propagate in different media, and the degree of shock wave attenuation will also vary. Here, the calculation of the attenuation characteristics of shock wave propagation overpressure under different conditions is used to describe the overpressure distribution within a certain distance after the explosion of the warhead. The existing computational models are mainly as follows:

$$\Delta P_+ = \frac{A}{\bar{r}} + \frac{B}{\bar{r}^2} + \frac{C}{\bar{r}^3} + D$$

In the formula, ΔP_+ —Peak value of shock wave overpressure;
 \bar{r} —Compare distance;
 A,B,C and D—Experimental fitting coefficient;

Obtain the shock wave overpressure on the surface of the component by measuring the distance between the geometric center of the component and the explosion point. By comparing the surface shock wave overpressure of the component with the damage threshold of the component shock wave, the calculation results of the damage effect of the component under the action of the shock wave are obtained.

3.2 Calculation of component damage effectiveness

Calculate the probability of damage to the target component based on the damage criterion function and hit damage element parameters. According to the Monte Carlo method, multiple samples of the end trajectory are taken to calculate the probability of damage to the component by the fragment damage element and shock wave damage element^[7]. The calculation result set $EI = \{eI1, eI2, \dots, eIn\}$ is obtained by sampling N times, and the expected EIT of the warhead's damage probability to the component is finally obtained. The calculation is shown in the following equation.

$$E_{IT} = \frac{1}{N} \sum_{i=1}^n E_{Ii}$$

3.3 Calculation of target destruction effectiveness

The calculation results of the damage effectiveness of each component of the target are weighted and summed according to the logical division of the damage tree and the weight of the component damage, and finally the calculation results of the damage effectiveness of each component in the target are obtained. Based on the target damage result set $EF=\{Ef1, Ef2, \dots, Efn\}$, combined with the damage trees corresponding to different levels of damage, calculate the overall target damage probability for that level of damage. According to the damage tree corresponding to different levels of damage, calculate the probability of sub events occurring layer by layer, and finally obtain the probability e of the ammunition warhead damaging the target.

4. Analysis method of destructive effect

Table4.1 .Characterization of destructive effects.

Damage effect	Damage probability
Mild damage	0~0.25
Moderate damage	0.25~0.50
Severe damage	0.5~1

Based on the calculation results of target damage effectiveness and the table above, determine the damage effect of the target under the given damage tree, and thus determine the degree of damage to the target function. For example, for armored vehicle targets, if the damage probability of their Level I damage tree is 0.15, it indicates that the damage tree is mildly damaged, revealing that the movement function of armored vehicle targets is mildly impaired.

5. Methods for characterizing destructive effectiveness

For different targets, the characterization of weapon and ammunition damage effectiveness is not the same, usually using ① single shot damage probability (hereinafter referred to as damage probability); ② Destruction of territory; ③ Number of damaged targets; ④ Percentage coverage of target area; ⑤ Use ammunition. Characterize the destructive effectiveness of weapons and ammunition using five characterization metrics^[8].

The characterization of destructive effectiveness depends on the method of ammunition delivery. When using a continuous firing method to strike a target, the aiming point is fixed, so the probability of single ammunition causing damage to the target is constant. Usually, the method of characterizing the damage effectiveness of a target is to calculate the amount of ammunition n required to cause certain damage to the target under continuous shooting conditions. The calculation method is as follows:

$$P_n = 1 - (1 - P_i)^n$$

In the formula, P_n -represents the probability of damage to the target caused by continuous firing of ammunition;

P_i - the probability of single shot damage to the target under a single shot;

Under the condition of expected damage probability P_n , the amount of ammunition used n can be obtained based on the known damage probability P_i of a single missile. Generally, the expected damage probability P_n is a number not greater than 1, and n can be calculated by the following equation.

$$n = \left\lceil \frac{\lg(1 - P_n)}{\lg(1 - P_i)} \right\rceil + 1$$

When using salvo to strike a target, there are usually multiple aiming points within a certain area. At this time, the calculation of the required amount of ammunition to achieve a certain degree of damage to the target is usually used as a method to characterize the effectiveness of the target's destruction. The calculation method is as follows:

By setting the aiming point area and calculating the probability of ammunition damage to the target, the probability of ammunition damage to the target under multiple aiming points can be obtained. Assuming that ammunition strikes are not independent and each launch vehicle has a different aiming point, the probability of damage to the target when multiple rounds explode can be calculated under different aiming points and projectile target intersection conditions, as follows:

$$P = 1 - \prod_{i=1}^n (1 - P_i)$$

In the formula, P_n - represents the probability of damage caused by ammunition to the target under multiple shots;

P_i - single shot damage probability of a single ammunition to a target;

n - The amount of ammunition used.

6. Conclusion

This article proposes a method for evaluating damage effectiveness, which includes methods for calculating missile target intersection, calculating damage effectiveness, analyzing damage effectiveness, and characterizing damage effectiveness. The calculation method for missile target intersection includes coordinate system transformation method and end trajectory calculation method. The calculation methods for damage effectiveness include component damage effect calculation, component damage effectiveness calculation, and target damage effectiveness calculation. Combined with practical use, good results have been achieved.

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