Self-calibration Algorithm of Kruppa Equation Based on Planar Motion Constraints

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Abstract. Camera calibration is a key technology in computer vision. Traditional methods rely on calibration objects or precise control of camera motion, which limits their application in unknown environments. Self-calibration techniques estimate camera intrinsic parameters using geometric constraints from multiple uncalibrated images. Methods based on Kruppa equations have been widely studied due to their rigorous mathematical foundation. However, these methods face challenges such as complex nonlinear solutions and sensitivity to noise in practical applications. This paper proposes an improved algorithm based on planar motion constraints, which transforms the traditional nonlinear optimization problem into solving a system of linear equations by analyzing the geometric properties of Kruppa equations under specific motion patterns, significantly reducing computational complexity. Experimental results show that this method accurately estimates camera intrinsic parameters in both simulated and real image sequences and demonstrates good robustness to noise.

Keywords: Camera self-calibration; Kruppa equations; Planar motion constraints.

1. Introduction

Camera calibration is a key technology in the field of computer vision, aiming to establish the geometric relationship between three-dimensional space points and their two-dimensional image projections, and provide basic support for tasks such as geometric measurement, three-dimensional reconstruction, and robot navigation [1]. Traditional camera calibration techniques can be mainly divided into two major categories: calibration methods based on reference objects and active vision calibration methods [2]. The former relies on calibration objects with known three-dimensional structures (such as checkerboards), while the latter requires precise control of the camera's motion trajectory. Although these methods perform well in controlled environments, their strong dependence on prior calibration objects or precise motion control severely restricts their application effectiveness in dynamic and unknown scenarios.

To break through the above limitations, the camera self-calibration technology has emerged as the times require. The so-called self-calibration refers to the process of determining the internal parameters of the camera directly through multiple uncalibrated images [3]. Different from traditional calibration methods, self-calibration does not rely on external references, nor does it require precise control of the camera's movement. It only needs to use the geometric consistency constraint relationship of corresponding points in multiple frames of images to complete the estimation of the camera's internal parameters. With its remarkable flexibility and practicality, this method shows unique advantages in practical applications. At present, most mainstream self-calibration methods are based on the absolute conic or its dual form [4,5]. Among them, the method based on the Kruppa equation has been widely studied due to its strict mathematical foundation. However, this method still faces two key challenges in practical applications: first, the implicit unknown scale factor in the equation requires solving through non-linear optimization, which has high computational complexity and is prone to falling into local optimal solutions; second, the calibration results are sensitive to image noise, and the robustness needs to be improved [6].

In response to the above problems, this paper proposes an improved algorithm based on planar motion constraints. By conducting an in - depth analysis of the geometric characteristics of the camera in a planar motion scenario, the solution of the unknown scale factor is ingeniously transformed into an eigenvalue problem. Thus, the originally complex non - linear optimization problem is successfully simplified into the process of solving a system of linear equations. This improvement not only significantly reduces the computational complexity but also enhances the practical application efficiency of the algorithm. In the experimental part, the effectiveness and accuracy of the proposed method are verified through simulation data and real image sequences, providing a new solution for camera self - calibration in dynamic scenarios.

2. Related Work

In the early 1990s, Faugeras et al. [3,7] first proposed a camera self-calibration method based on the Kruppa equations. By introducing the concept of the projected image of the absolute conic and combining the geometric properties of epipolar transformation, they established a complete mathematical framework for the Kruppa equations. Lourakis et al. [8] simplified the original Kruppa equations using the singular value decomposition (SVD) form of the fundamental matrix. Zeller et al. [9] took a different approach by transforming the solution of the Kruppa equation into a mathematical programming problem. They used the Levenberg-Marquardt optimization algorithm [10] to solve for camera parameters by minimizing the distances between feature points in multiple images and their corresponding epipolar lines. Although this method reduces the difficulty of the solution to some extent, it still has inherent drawbacks such as unstable convergence and a tendency to fall into local optimal solutions, which limit its performance in practical applications. Triggs [11] applied the absolute dual quadric to the self-calibration technique. First, a projective reconstruction of the image sequence was carried out in advance to transform the internal parameter constraints into absolute quadric constraints. Then, the sequential quadratic programming (SQP) algorithm was used to directly solve the minimum value of the cost function under matrix constraints, and thus the calibration work was completed.

However, in essence, all these algorithms need to solve nonlinear equations or nonlinear programming problems. They not only have high computational complexity but are also extremely sensitive to the selection of initial values. In addition, under noise interference, the robustness of the algorithms decreases significantly, and they are prone to getting trapped in local optimal solutions [12]. The main root of these problems lies in the inherent unknown scale factor in the Kruppa equations, which makes it difficult to avoid nonlinear optimization in the calibration process. To address this difficult problem, Ma et al. [13] analyzed the singularity and solvability of the Kruppa equations and proved that in the case of special motion, the Kruppa equations can be normalized into linear equations, and the camera intrinsic parameters can be solved linearly, reducing the difficulty of the solution. Wang [14] studied the special scenario of circular motion and proposed a new linear solution method. By analyzing the geometric characteristics of the camera during circular motion, this method successfully derived the analytical expressions of the unknown coefficients in the Kruppa equations, enabling the solution of the camera intrinsic parameters to be transformed into the calculation of a system of linear equations.

3. Linear Solution of Planar Motion Constraints of Kruppa Equations

3.1 Camera Model

The camera model is the core mathematical model in computer vision. It precisely describes the geometric mapping relationship between the three-dimensional world coordinate system and the two-dimensional image coordinate system. According to the theory of projective geometry, this model establishes a complete projective transformation relationship between the spatial object points and their image projection points [15]. Specifically, given an arbitrary three - dimensional point $\boldsymbol{X} = [X_W \quad Y_W \quad Z_W \quad 1]^{\mathrm{T}}$ in the world coordinate system, the corresponding image point is

$\boldsymbol{x} = [u \quad v \quad 1]^{\mathrm{T}}$, and the relationship between these two points can be represented by the projection matrix $\boldsymbol{P}$:

$$Z_C \boldsymbol{x} = \boldsymbol{PX} \tag{1}$$

Among them, $\boldsymbol{P}$ represents the projection matrix of the camera, and Z_C is an unknown scale factor. According to Richard's projection matrix model, the camera's projection matrix $\boldsymbol{P}$ contains information about the camera's internal and external parameters and can be decomposed into [16,17]:

$$\boldsymbol{P} = \boldsymbol{K}\left[\boldsymbol{R} \mid -\boldsymbol{R}\tilde{\boldsymbol{t}}\right] \tag{2}$$

In the formula: the matrix $\boldsymbol{K}$ represents the camera calibration matrix, $\tilde{\boldsymbol{t}}$ indicating the coordinates of the camera center in the world coordinate system, and $\boldsymbol{R}$ is the rotation matrix representing the direction of the camera coordinate system. The specific expression of the calibration matrix $\boldsymbol{K}$ is as follows:

$$\boldsymbol{K} = \begin{bmatrix} f_x & \gamma & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

The parameters included in $\boldsymbol{K}$ are collectively referred to as camera intrinsic parameters. Among them, f_x and f_y represent the equivalent focal lengths in the x and y directions of the image plane coordinate system respectively, with the unit of pixels; $(x_0, y_0)^{\mathrm{T}}$ is the pixel coordinate of the principal point; and γ is the distortion parameter.

3.2 Solve the Calibration Matrix

For most standard cameras, the distortion parameter γ is zero. Therefore, the core problem of the camera self - calibration research in this paper is simplified to solving four key intrinsic parameters: f_x, f_y, x_0, and y_0. The solved calibration matrix can be simplified into the following form:

$$\boldsymbol{K} = \begin{bmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Assume that the camera's calibration matrix $\boldsymbol{K}$ remains unchanged. Considering an image pair of the same scene captured from two different viewpoints, we can establish an epipolar geometric constraint relationship through the Image of the Absolute Conic (IAC). Based on this, the Kruppa equations in matrix form can be derived (the specific derivation method can be referred to in references [13,18,19]). Its form is as follows:

$$\boldsymbol{FCF}^{\mathrm{T}} = \lambda [\boldsymbol{e}']_{\times} \boldsymbol{C} [\boldsymbol{e}']_{\times}^{T} \tag{5}$$

In the formula: $\boldsymbol{F}$ represents the fundamental matrix between the two images, λ is an unknown positive proportional factor, $\boldsymbol{C} = \boldsymbol{KK}^{\mathrm{T}}$. $\boldsymbol{e}' = (e_1 \quad e_2 \quad e_3)^{\mathrm{T}}$ is the epipole in the second image, and $[\boldsymbol{e}']_{\times}$ is an antisymmetric matrix defined in the following form:

$$[\boldsymbol{e}']_{\times} = \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix} \tag{6}$$

It can be seen that the matrix $\boldsymbol{C}$ is symmetric. To simplify the expression, we introduce the following notation:

$$\boldsymbol{C} == \begin{bmatrix} f_x^2 + x_0^2 & x_0 y_0 & x_0 \\ x_0 y_0 & f_y^2 + y_0^2 & y_0 \\ x_0 & y_0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_2 & c_4 & c_5 \\ c_3 & c_5 & c_6 \end{bmatrix} \tag{7}$$

Let $\boldsymbol{c} = [c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6]^T$. Then, we can rewrite the constraint provided by Equation (5) in the form of a homogeneous linear equation:

$$\boldsymbol{A}_{(\lambda)}\boldsymbol{c} = \boldsymbol{0} \tag{8}$$

Where the elements in matrix $\boldsymbol{A}_{(\lambda)}$ are expressions of λ . In the traditional solution of the Kruppa equations, the uncertainty of the scale factor λ makes it necessary to perform non - linear optimization. However, when the camera moves in a plane (i.e., the camera coordinate system only translates along the X - Z plane and rotates around the Y - axis), this special motion mode can bring a crucial simplification to the calibration problem. According to the theoretical analysis by Ma et al. [12], under planar motion, the square root of the proportional factor λ corresponds to one of the two non-zero eigenvalues of the fundamental matrix $\boldsymbol{F}^T[\boldsymbol{e}']_\times$. However, due to the inability to directly determine which non-zero eigenvalue is valid, it becomes necessary to consider all possible 2^n combinations when n fundamental matrices are provided. This combinatorial challenge arises from the inherent ambiguity in eigenvalue selection under such motion constraints.

Generally speaking, the Kruppa equation corresponding to each fundamental matrix can provide two independent constraints on the unknown vector $\boldsymbol{c}$. Since the vector $\boldsymbol{c}$ contains six unknown parameters, at least three Kruppa equations are required to construct a complete system of constraint equations. It is worth noting that there are two possible values for the scale factor λ corresponding to each fundamental matrix (i.e., the two non - zero eigenvalues of the matrix $\boldsymbol{F}^T[\boldsymbol{e}']_\times$). Therefore, when three fundamental matrices are used, it is necessary to examine $2^3=8$ possible parameter combinations. For each possible parameter combination, we first solve the system of linear equations $\boldsymbol{Ac} = \boldsymbol{0}$ to determine the value of the vector $\boldsymbol{c}$, and then use the vector $\boldsymbol{c}$ to recover the corresponding matrix $\boldsymbol{C}$. However, among these eight combinations, only the matrix $\boldsymbol{C}$ obtained from one combination is positive - definite. Finally, through the Cholesky decomposition [20] of the matrix $\boldsymbol{C}$, the required calibration matrix $\boldsymbol{K}$ is extracted.

4. Experimental Results

In this section, we conducted experiments on simulated data and real image sequences respectively to verify the effectiveness of the proposed method. The hardware configuration used in the experiments was the same computer, equipped with an Intel(R) Core(TM) i9 - 14900HX 2.20 GHz processor and running MATLAB.

4.1 Simulation Experiment

In the simulation experiment section, we first randomly generate a set of spatial points in a three - dimensional space as the calibration scene. To simulate the planar motion constraint conditions, the virtual camera is set to follow the planar motion mode: it only rotates around the Y - axis and translates within the X - Z plane. By controlling the camera to observe the spatial points from three different viewpoints (all satisfying the above - mentioned motion constraints), the three - dimensional point set is projected onto the virtual imaging plane to obtain the corresponding two - dimensional image coordinates. Based on these three sets of image corresponding points that satisfy the planar motion constraints, the Kruppa equation is constructed and the calibration matrix of the virtual camera is solved. The ground truth of the set calibration matrix $\boldsymbol{K}$ is:

$$\boldsymbol{K} = \begin{bmatrix} 800 & 0 & 320 \\ 0 & 790 & 240 \\ 0 & 0 & 1 \end{bmatrix}$$

In order to measure the accuracy and robustness of the method proposed in this chapter under different image noise levels, this study designed a progressive noise intensity test experiment. By adding Gaussian noise of different intensities (the noise level increases from 0 pixels to 1 pixel) to the images, the possible image noise interference in real scenarios was simulated. 500 independent repeated experiments were conducted at each noise intensity level, and the average value of the absolute error was calculated as the final performance index. As can be seen from Figure 1, when a certain amount of Gaussian noise is added to the image points, the method in this paper can still maintain a high calibration accuracy, and the internal parameter estimation error is within a reasonable range.

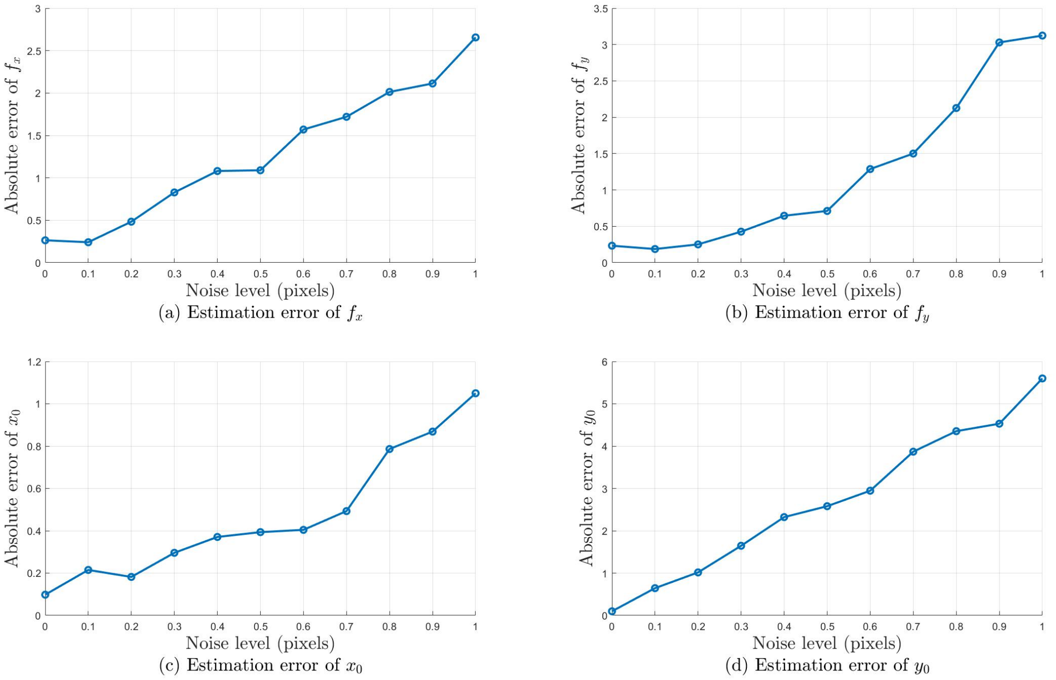

Fig. 1 Curve of absolute error of internal parameter estimation values varying with noise

4.2 Image sequence experiment

In the image sequence experiment, to evaluate the calibration performance of our method, we conducted comparative experiments using a standard chessboard calibration plate, as shown in Figure 2. During the image acquisition process, the calibration plate remained stationary, with the camera strictly restricted to translational movement within the X-Z plane and rotation around the Y-axis, ensuring compliance with the planar motion constraints proposed in this paper. Based on this, by adjusting the camera's pose, images were taken from three different angles.

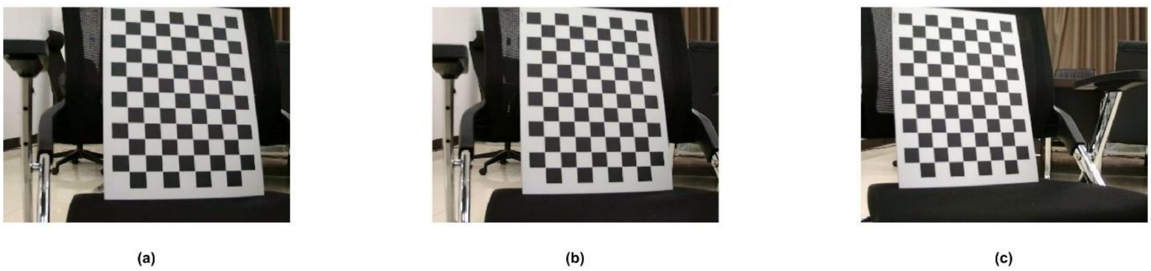

Fig. 2 The pictures used in the experiment

In practical applications, since it is difficult to directly obtain the true values of camera intrinsic parameters, in order to objectively evaluate the calibration accuracy of the method proposed in this paper, we selected the Zhang Zhengyou calibration method, which is widely recognized in the field of computer vision, as the reference method for comparative analysis. The Zhang Zhengyou calibration method was proposed by Zhang et al. [21]. It is currently one of the recognized standard camera calibration algorithms, with high authority and universality. Table 1 shows the comparison of the calibration results of the two methods. Table 1 presents a comparison of the calibration results between the two methods.

Table 1. Result comparison

Camera intrinsic parameters	Zhang Zhengyou calibration method	Our method
f_x	1011.58	1010.50
f_y	1012.14	1023.32
x_0	428.02	431.23
y_0	276.41	264.23

It can be seen from Table 1 that the calibration results of the method in this paper are in good agreement with the classic Zhang Zhengyou calibration method.

5. Conclusion

This paper addresses the issues of high computational complexity and noise sensitivity in the traditional self - calibration method based on the Kruppa equations and proposes an improved algorithm based on planar motion constraints. By leveraging the geometric properties of planar motion, the solution of the scale factor is transformed into an eigenvalue problem, achieving a linear solution for the non - linear problem and significantly improving the computational efficiency. Experimental verification shows that in the simulation environment, when the image noise level reaches 1 pixel, the internal parameter estimation error remains within 3%. In real - image experiments, the difference from the results of Zhang Zhengyou calibration method is less than 5%, fully verifying the effectiveness of the method. The theoretical contribution of this research lies in revealing the simplified solution mechanism of the Kruppa equations under planar motion constraints, providing a new idea for self - calibration in dynamic scenes. Future research will focus on exploring robust calibration methods under non - strictly planar motion conditions and further developing adaptive optimization strategies that can adapt to higher - noise environments to enhance the applicability and stability of the algorithm in real complex scenes.

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