

Optimal Airport Resource Allocation for Security Check Based on Queuing Theory

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Abstract. We study the efficiency of security check and propose an optimal method about airport resources for security check in this paper. First, we examine the process of security check at airports, and utilize the queuing theory to construct the queuing model and calculate the relevant indicators of the queuing system of airport security. Second, we establish the objective programming model of open security checkpoints, whose objective function is the profit of security system in airports. Then, we design the algorithm to solve the above models and conduct the numerical analysis. Finally, we conduct numerical analysis to study optimal airport resource allocation for security check. We show that the objective programming model is simple to calculate, and has practical application value for solving the problem of allocation of the number of security checkpoints at airports and airlines.

Keywords: airport security check; security checkpoint; queuing theory; objective programming; optimization.

1. Introduction

With the rapid development of the air transport industry, airports have gained great benefits but also face great challenges at the same time. As the key node in the global transport network, airports' operational efficiency has a significant impact on the passenger experience and economic benefits. Due to the increase in passengers, airport security resources are also more strained, which may lead to adverse consequences such as too long waiting time for passengers, detention at airports, and decline in service quality. With the acceleration of the process of globalization, security check in airports that is an important barrier to ensure aviation safety become increasingly prominent. The boarding process of passengers generally includes arrival at airports, check-in, security check, waiting, and departure, among which security check are more complicated and the bottlenecks in airports. The efficiency of security check often affects the smooth travel of passengers and service quality. As a part of the airport service, security check is the typical multi-server queuing model where the queuing theory has been widely used. As a mathematical theory to study the phenomenon of waiting line, queuing theory plays an important role in the optimization of airport security resources.

As a key link in aviation security, airport security check directly affects the operation of the airport and the travel experience of passengers, thus attracting extensive attention from scholars. Donmez [1] introduces a decision support system called Airport Ground Optimizer and demonstrates its strong reliability in the practical optimization application of security check research elements. Walter et al. [2] construct a more general linear regression model based on a constant ratio to describe the relationship between design hour loads and annual demand, and provide guidance for airport planning. Wu and Xie [3] introduce a dynamic strategy, the Join the Shortest Queue strategy, to improve load balancing in the airport security check process. Skorupski et al. [4] aim to analyze the throughput of the baggage security check system and serve as an auxiliary tool for airport management in solving operational problems. Knol et al. [5] identify and quantify the relationship and trade-offs between the performance of illegal item detection and the average queuing time at airport security checkpoints through a cognitive agent model simulating the operation of airport security checkpoints. Kierzkowski and Kisiel [6] describe in detail the structure,

process, characteristics, and parameterization process of the security checkpoint simulation model in the context of the pandemic, providing valuable references for optimizing the operation of airport security checkpoints.

In terms of the various situations faced by passengers in queuing, scholars have conducted detailed analyses. De Lange et al. [7] consider the application of virtual queuing principles in airport security channels. Kierzkowski and Kisiel [8] propose a dynamic management model for airport check-in counter systems. Chiti et al. [9] develop an integrated service software platform aimed at predicting waiting times at security counters and the number of active security control counters required to improve overall efficiency based on queuing theory analysis methods. Li et al. [10] explore which queuing structure is efficient under certain conditions without reducing the safety level and examine the relationship between passenger strategies and the efficiency of different queuing structures. Pala and Zhuang [11] construct a two-dimensional Markov chain model based on game theory to study the average waiting time under the influence of impatient applicants. Rodríguez-Sanz et al. [12] introduce machine learning to capture and analyze queuing behavior patterns. Yi et al. [13] provide a more realistic and detailed explanation framework for pedestrian queuing behavior numerically. Li et al. [14] investigate the impact of environmental factors and customer personal factors on the choice of security channels. Forbes et al. [15] obtain more stable and accurate solutions to stochastic problems by running simulations on each candidate solution of the main problem.

Although the above literature all designed queuing mechanisms for the security check process at airports using queuing theory methods, they did not consider the operating costs of the airport security system and could not simultaneously improve system efficiency and reduce costs. Different from the above literature, we study the efficiency of security check and propose an optimal method about airport resources for security check in this paper. First, we examine the process of security check at airports, and utilize the queuing theory to construct the queuing model. Second, we establish the objective programming model of open security checkpoints. Then, we design the algorithm to solve the above models and conduct the numerical analysis. Finally, we conduct numerical analysis to study optimal airport resource allocation for security check.

2. Model

2.1 Problem Description

Security check at airports is set up to prevent passengers from carrying items that may endanger the safety of the aircraft, which consist of two parts: security check for passengers and carry-on baggage. Security check for passengers at airports includes ticket checks, body checks and boarding from the moment passengers arrive at airports to the end of the departure. We consider airports as system capacity, and ticket counters and security checkpoints as service institutions. In the queuing system, a passenger's service time is from the moment he enters the area of ticket checks to the time he leaves the area of security check, the waiting time is the average waiting time for the passenger to queue at the area, the average service time is the difference between the dwell time and the waiting time, and the average length is the throughput of security check at airports. The throughput of security check refers to the number of passengers who can pass through the area of security check per unit time under certain service levels, which is a performance indicator that reflects service quality of security check and is a measure of service level at airports. We assume that the number of passengers arriving at airport security checkpoints is independent and follows a Poisson distribution at different times. For security checkpoints, the service modes are first come and first served, the service time follows a negative exponential distribution, and the service rates are the same. Due to the abundant space at airports, we assume that the capacity of the security check system is infinite and thus use the queuing model to analyze the throughput of security check at airports. The model parameters and variables are listed in Table 1.

Table 1. The model parameters and variables

Notations	Description
Parameters	
λ	Average arrival rate
μ	Average service rate
N	Number of security checkpoints
P_{\min}	Minimum probability of busy security checkpoints
P_{\max}	Maximum probability of busy security checkpoints
P_s	Revenue earned by a passenger during airport security check
c_s	Unit cost of opening a security checkpoint at an airport
q_w	Proportional coefficient related to the value-added income of passenger satisfaction
Decision variables	
c	Number of open security checkpoints
Dependent variables	
ρ	Service intensity
L_s	Average captain
L_q	Average queue length
W_s	Average waiting time
W_q	Average queuing time
l_c	Standard queue length
P_n	Probability of busy security checkpoints
π	Net revenue of airport security check system

2.2 Queuing Model

The queue system of $M / M / c$ is more optimized than that composed of $c M / M / 1$, so we assume that the security check at airports is a queue system of $M / M / c$. In the queue system of $M / M / c$, the service intensity is

$$\rho = \frac{\lambda}{c\mu}. \tag{1}$$

The probability that a passenger arrive at the queuing system of the security check at airports without queuing is

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda / \mu)^n}{n!} + \frac{(\lambda / \mu)^c}{c!} \cdot \frac{1}{1-\rho} \right]^{-1}. \tag{2}$$

The average queue length is

$$L_q = \frac{\rho(\lambda / \mu)^c}{c!(1-\rho)^2} P_0. \tag{3}$$

According to the Little formula, the average queue time

$$W_q = \frac{L_q}{\lambda}. \tag{4}$$

The probability that a passenger needs to queue at an airport security checkpoint is equal to the average probability of busy security checkpoints, that is

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & (n=0,1,\dots,c) \\ \frac{(\lambda/\mu)^n}{c!c^{n-c}} P_0 & (n \geq c) \end{cases} \quad (5)$$

The service level of an airport is generally required to meet certain standards. We adopt the airport facility service time standards formulated by the International Air Transport Association (IATA), as shown in Table 2.

Table 2. The airport facility service time standards formulated by IATA

Server	Min-acceptable time (minutes)	Max-acceptable time (minutes)
Economy class check-in	0-12	12-30
Business class check-in	0-3	3-5
Arrive at passport control	0-7	7-15
Departure passport control	0-5	5-10
Baggage claim	0-12	12-18
Security	0-3	3-7

The average wait time is

$$W_s = W_q + \frac{1}{\mu} \quad (6)$$

Let $W_s = W_{IATA}$, we obtain the number of open security checkpoints, where W_{IATA} is the airport facility service time standards formulated by IATA. When calculating the number of open security checkpoints, we must solve a nonlinear equation and cannot obtain a general formula for the solution. However, we can use numerical solutions such as the graphical method. First, we can use Eq. (3) to draw the curve of the average queue length and the number of open security checkpoints. Then, we find the intersection of the curve and the horizontal straight line l_c , which is the number of open security checkpoints. l_c can be determined according to the above standards by the following formula

$$l_c = \lambda(W_{IATA} - \frac{1}{\mu}) \quad (7)$$

2.3 Objective Programming Model

We take the maximum profit of the security check system as the objective function, that is, the sum of the revenue generated by the security check system serving passengers and the value-added revenue brought by satisfying passengers, minus the operating cost of opening security checkpoints. Thus, the objective programming model is

$$\begin{aligned} \max \pi &= p_s P_n c \mu + q_w (l_c - L_q) - c_s c \\ \text{s.t.} &\begin{cases} L_q \leq l_c \\ c \leq N \\ c > \rho \\ P_{\min} \leq P_n \leq P_{\max} \end{cases} \end{aligned} \quad (8)$$

where the first constraint is that the average queue length cannot exceed the standard length so that airports receive the value-added revenue from passenger satisfaction. If so, passenger satisfaction and service levels will be lower, and airports' reputation may be damaged. The second constraint is the limitation of airport security resources, i.e., the number of open security checkpoints cannot exceed the number of security checkpoints. The third constraint is that the number of open security checkpoints cannot be less than the average number of checkpoints used, which is also the constraint that the queue length in Eq. (3) cannot be zero. Because according to the queuing theory,

if service intensity of the security checkpoint is close to 1, the queue length may be infinite, which means that the probability of busy security checkpoints is close to 1. The fourth constraint is that the probability of busy security checkpoints should be a reasonable range. Because a too high probability may cause security inspectors tired easily and low efficiency, and a too low probability may result in low resource utilization of security check.

3. Algorithm

Eq. (8) is a nonlinear integer programming model that is difficult to solve directly. However, since the number c may vary from a small range, we use the combination algorithm of heuristics and enumeration here. According to the marginal analysis, the optimal number c^* corresponding to the maximum objective function should satisfy

$$\begin{aligned} \frac{c_s}{\mu p_s} &\leq \frac{q_w}{p_s} \rho(W_q(c^*-1) - W_q(c^*)) - c^*[P_n(c^*-1) - P_n(c^*)] + P_n(c^*-1) \\ \frac{c_s}{\mu p_s} &\geq \frac{q_w}{p_s} \rho(W_q(c^*) - W_q(c^*+1)) - c^*[P_n(c^*) - P_n(c^*+1)] + P_n(c^*+1) \end{aligned} \quad (9)$$

Then, we check whether the optimal number c^* from Eq. (9) satisfies the constraints of Eq. (8). If satisfied, the optimal number c^* is the optimal solution; If not, we check which constraint is not satisfied. Then, according to that the length L_q and the probability P_n decreases with the number c , we can obtain the optimal solution by adding or subtracting 1 to the optimal number c^* .

Because the optimal number c^* should be in the vicinity of service intensity ρ , we can respectively obtain the probability P_n and the length L_q according to Eq. (3) and (5). Then, we substitute them to the right side of Eq. (9) and judge when the number c may satisfy Eq. (9), so that we obtain the optimal number of security checkpoints.

4. Numerical Analysis

We conduct numerical analysis in this section. For an airport, we assume that $\lambda = 3.1$ person/minute, $\mu = 0.5$ person/minute, $p_s = 10$ RMB/person, $c_s = 5$ RMB/minutes, $q_w = 0.5$ RMB/minutes, $P_{\min} = 0.45$, and $P_{\max} = 0.75$. If $W_s = W_{ATA} = 10$ minutes, then the length $l_c = 15 \times 1.9 = 28$. Next, we discuss when Eq. (9) will hold.

Based on the above information, we can obtain $\rho = 3.8$ and assume $c = 3, 4, 5, 6$. Then, the left side of Eq. (9) is $c_s/(\mu p_s) = 5/(0.5 \times 10) = 1$, and the coefficient of the first term on the right side is $\rho q_w/p_s = 3.8 \times 0.5/10 = 0.19$. According to Eq. (3) and (5), we can obtain the probability P_n and the length L_q , as shown in Table 3. We can further obtain the optimal solution $c^* = 22$ that satisfies Eq. (9). However, the probability is 0.89 at this time, which does not satisfy the range of the probability of busy security checkpoints.

In order to satisfy all the constraints of Eq. (8), we assume the optimal number $c^* = 23$. Then, we obtain the probability $P_n = 0.48$, the length $L_q = 1.52$, the time $W_q = 0.8$, and the objective function of Eq. (8) is -13.76 RMB. Moreover, we obtain that the maximum net income is $14 - 13.76 = 0.24$ RMB by adding $q_w l_c = 0.5 \times 28 = 14$ RMB. If we adjust the probability to $[0.45, 0.9]$, the optimal number is $c^* = 22$. Then, we obtain that the maximum objective function is -11.53 RMB and the maximum net benefit is 2.47 RMB.

Table 3. The range of various optimal solutions

c	$P_n(c)$	$P_n(c-1) - P_n(c)$	$P_n(c) - P_n(c+1)$	$L_q(c)$	$W_q(c)$	$W_q(c-1) - W_q(c)$	$W_q(c) - W_q(c+1)$	The right side of Eq. (9)
21	1*	-	0.11*	∞	∞	-	∞	-
22	0.89	0.11*	0.41	16.94	8.92	∞	8.92	$\infty, 0.54$
23	0.48	0.41	0.22	1.52	0.80	8.12	0.58	0.38,-0.73
24	0.26	0.22	-	0.41	0.22	0.58	-	-

5. Conclusion

In this paper, we construct a queuing model for the security check process, and analyze the three performance indicators of the airport security system, the average queue length, the average queuing time and the probability of passengers arriving in the queue, so as to provide an important reference for airports to improve passenger satisfaction. The objective programming model is simple to calculate, and has practical application value for solving the problem of allocation of the number of security checkpoints at airports and airlines. We will focus on the optimization of the security check process, and further propose the methods of airport security optimization in the future.

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