# Decoding Historical Stair Usage through Wear Analysis

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**Abstract.** Stairs are durable building components that maintain the presence of past human activities by displaying signs of wear and tear. Understanding the dynamic trends in historical usage is challenging due to the intricate interplay between mechanical wear, environmental conditions, and incomplete historical documents. To gain a deeper understanding of the usage of historical staircases, this study innovatively adopts a data-driven approach, combining Archard's wear law with convolutional neural networks (CNN), introducing a non-destructive hybrid measurement model that integrates 2D and 3D imaging and mechanical principles, and training CNN with multi angle images to generate wear heat maps for accurate prediction of regional wear; Using Archard's wear law and Poisson distribution to model the relationship between total wear and fine wear, infer historical pedestrian flow intensity, and then analyze directional preferences; Using a continuous force distribution model to divide the staircase area and comparing wear patterns to distinguish between single and double user scenarios to estimate group size; Construct an age estimation model based on wear and tear, taking into account various influencing factors. Overall, the method used in this article not only significantly improves the accuracy of wear analysis, but also provides a scientific and efficient new analytical tool for archaeological research and architectural heritage protection.

**Keywords:** Archard’s wear law; CNN-based wear quantification; Poisson usage modeling; Probabilistic age estimation; Archaeological crowd analysis.

## 1. Introduction

Stairs are durable building components that maintain the presence of past human activities by displaying signs of wear and tear. Understanding the dynamic trends in historical usage is challenging due to the intricate interplay between mechanical wear, environmental conditions, and incomplete historical documents.

To gain a deeper understanding of the usage of historical staircases, this study innovatively adopts a data-driven approach, combining Archard's wear law with convolutional neural networks (CNN), introducing a non-destructive hybrid measurement model that integrates 2D and 3D imaging and mechanical principles, and training CNN with multi angle images to generate wear heat maps for accurate prediction of regional wear; Using Archard's wear law and Poisson distribution to model the relationship between total wear and fine wear, infer historical pedestrian flow intensity, and then analyze directional preferences; Using a continuous force distribution model to divide the staircase area and comparing wear patterns to distinguish between single and double user scenarios to estimate group size; Construct an age estimation model based on wear and tear, taking into account various influencing factors. Overall, the method used in this article not only significantly improves the accuracy of wear analysis, but also provides a scientific and efficient new analytical tool for archaeological research and architectural heritage protection.

## 2. Literature Review

The wear and friction properties of materials are essential in engineering, particularly in construction and machinery. This review summarises key findings from five studies on material wear behaviour. The model developed highlights how contact area and load influence wear, with contact area increasing with load under both elastic and plastic deformation [1]. Quartz content and grain size in granite significantly affect wear resistance, with higher quartz content improving resistance as found Hardness is a better indicator of wear resistance than Knoop micro-hardness [2]. During stair descent, friction demands are similar to level walking, and appropriate footwear mitigates safety risks, as shown by Older adults adopting safer strategies, such as lower ground reaction forces[3]. Illumination has minimal impact on friction demands. In Turkey, the natural building stones’ wear resistance is linked to density, porosity, and water absorption, as discovered. High porosity and water absorption reduce wear resistance[4]. Uniaxial compressive strength also correlates with wear resistance. Different contact loads affect stone flooring materials’ wear rates, as found[5]. Carbonate stones show linear wear with load, while granite exhibits nonlinear behaviour at high loads. Load changes can alter wear resistance rankings. Low loads cause three-body abrasion, while high loads lead to two-body abrasion, reducing wear rates.

## 3. Model Design and Analysis

### 3.1 Assumptions and Notations

To simplify the analysis of the problem, we have made the following assumptions.

Assumption 1: Forces exerted by pedestrians are assumed to be evenly distributed across stair elements.

Assumption 2: Short-term fluctuations in humidity and temperature are ignored, focusing on long-term average wear rates.

Assumption 3: Edge regions are exclusively worn by ascending traffic, while center regions experience bidirectional wear. The main variables used in this paper are described in Table 1.

Table 1: Symbol Description

| Symbol | Description | Unit |
|---|---|---|
| Vtotal | Total wear volume | cm3 |
| v | Wear volume caused by one traversal | cm3 |
| K | Wear coefficient (material-dependent) | Dimensionless |
| F | Normal force per pedestrian | N |
| H | Material hardness | N/mm2 |
| T | Stairwell age | years |

### 3.2 Wear Volume Measurement

#### 3.2.1 Measurement Using CNN

CNNs enable us to measure and predict the wear experienced on stairs with precision. Detailed annotations are made to a dataset that features photos of uneven stair surfaces, which are used in training the model. Identifying wear-related traits and improving prediction accuracy is the main objective of the CNN as the dataset expands. Application of CNNs to image processing involves many use cases, particularly in classification and segmentation methods. Complex structures such as convolutional, pooling, and fully connected layers are commonly used. With convolutional layers applied, features such as surface cracking or erosion can be extracted from the input images using filters. Essential details remain even as layer pools are used to compact the spatial features of these feature maps. After that, completely connected layers analyze the removed attributes to forecast wear volumes or gauge usage. An illustration of the CNN training process is shown in Figure 1.

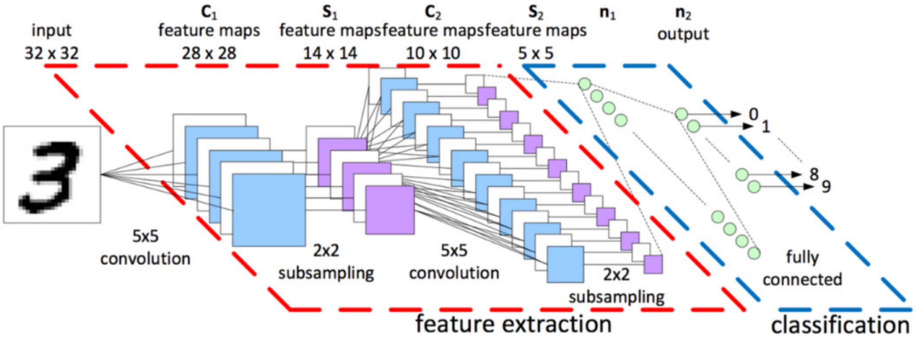


Figure 1: CNN Model Training Process

### 3.2.2 Integration with Stair Geometry

To provide comprehensive data, images are captured from multiple angles—front-facing, side, and 45-degree views--with consistent lighting to reduce shadow interference and emphasize wear patterns. The dataset is divided into training, validation, and testing sets for model development. Using a stochastic gradient descent (SGD) optimizer, the model adjusts filter weights iteratively to minimize the loss function, enhancing prediction accuracy. The trained model can then predict wear volume in different regions, enabling quantification of wear across the entire stair surface. Examples of the captured images are shown in Figure 2. In addition to 2D images, 3D scans of stair surfaces are incorporated to enhance the wear prediction model. These scans provide detailed surface maps, allowing the model to analyze both visual and topographical changes caused by wear. Figure 3 demonstrates a 3D surface scan of the stairs.

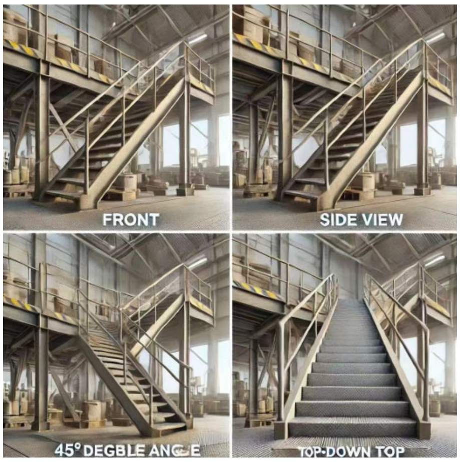


Figure 2: Stair images captured from various angles

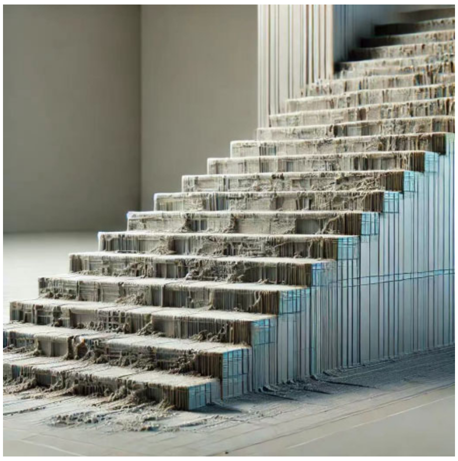

Figure 3: 3D Surface Scan of Stairs

### 3.2.3 Wear Volume Prediction

Once trained, the CNN model generates wear volume heatmaps, highlighting regions with higher wear intensity. This method provides detailed insights for maintenance planning. The heatmap is visualized in Figure 4. The total wear volume, $V_{\text{total}}$, is computed by summing the wear volumes of individual elements, $V_{\text{i}}$ :

$$V_{\text{total}} = \sum V_{\text{i}} \approx V_0 \times N_{\text{total}} \quad (1)$$

where each element represents 1 $\text{cm}^2$ , and $N_{\text{total}}$ denotes the total number of elements or the stair's surface area.

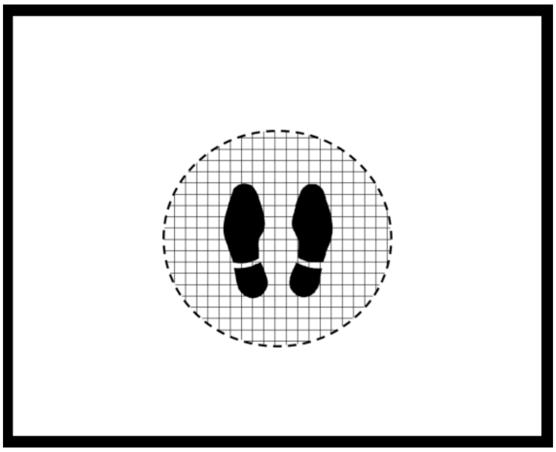

Figure 4: Stair images divided into elements

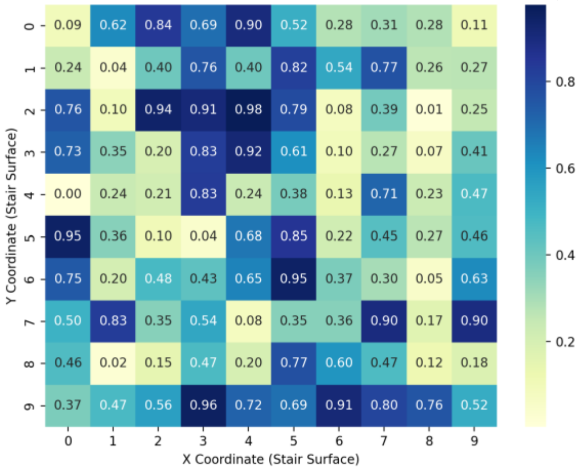


Figure 5: Wear Volume Heatmap

## 3.3 Basic Predictions from Stair Wear Patterns

### 3.3.1 Archard's Wear Law

To model the wear of the staircase, we apply Archard's wear law, which provides a relationship between the wear volume of a material and the mechanical parameters involved in the contact process. And we first calculate the wear volume in one element and then sum it up into the total wear volume of the whole stair.

For one single element, Archard's law is given by the equation:

$$v_{\mathbf{i}} = \frac{K \cdot f \cdot \mathbf{1}}{H} \tag{2}$$

where $v_{\mathbf{i}}$ is the wear volume, representing the amount of material removed from the surface. k is the wear coefficient, a material-dependent constant that characterizes the susceptibility of a material to wear under specific conditions. f is the normal force applied between two surfaces.

In contact, which in this case represents the weight of one element borne. h is the hardness of the material (typically measured in newtons per square millimeter or pascals, the common units being Vickers hardness or Brinell hardness). The sliding distance in classical theory is omitted here since the size of single element is already be defined in the initial measurement.

Each small element on the floor experiences varying levels of contact force depending on the pattern of use and the interaction with pedestrians. For each element, the local wear volume can be calculated by determining the local contact force f as constant.

The contact force f of one element depends on factors such as the weight of the user (denoted by F) and the distribution of pressure during each step. For example, a person's foot may exert a concentrated force in the middle of the tread or a more distributed force depending on how they place their foot. In the case of multiple individuals walking side by side, this could further increase the contact pressure in the center of the stair, as compared to the edges, resulting in differential wear patterns. However, in this case, we assume elements in contact with the pedestrians are evenly distributed, and the force one element bears is:

$$\frac{F}{N} = f \tag{3}$$

where $N$ is the number of elements worn in one traversal. By applying Archard's wear law to each element on one stair surface, the total wear volume of the stair by one traverse(v) can be calculated.

$$v = \sum v_{\mathbf{i}} = \sum \frac{K \cdot f \cdot 1}{H} \approx \frac{K \cdot F}{H} = \frac{K \cdot f \cdot N}{H} \tag{4}$$

### 3.3.2 The frequency of using stairs

We assume that the load force and sliding distance for each person remain approximately constant, based on an average person's weight and typical gait. We then use Archard's wear law to link the wear volume per individual use to the overall wear that is observed.

Let $V_{\text{obs}}$ be the total observed wear volume over the specified period T. Suppose each individual causes a local wear volume $V_{\text{stair, single}}$ = KFN, where K is the wear coefficient for the stair material, F is the normal force applied by one person, and N is the number of elements associated with a single traversal of the step. If the number of visitors in the time interval T is denoted by Q, then the total wear volume $V_{\text{total}}$ from all visitors can be approximated by

$$V_{\text{total}} \approx Qv. \quad (5)$$

We model the number of visitors N using a Poisson distribution with parameter λ, which is the average arrival rate (visitors per unit time). Thus, N follows:

$$P(N = k) = \frac{(\lambda T)^k e^{-\lambda T}}{k!} \quad (6)$$

The expected value of *N* is $E[N]$ = λT. Therefore, the expected total wear volume over the interval T can be written as:

$$V_{\text{total}} = E[Q]v = (\lambda T)(\frac{K \cdot F}{H}) \quad (7)$$

Since $V_{\text{total}}$ is the total actual measured volume lost, one can solve for $\lambda$ by equating the observed wear to its expected value, yielding

$$\lambda = \frac{V_{\text{total}} \cdot H}{TKF} \quad (8)$$

In practice, if the total observed wear volume arises from multiple steps, one would sum or otherwise combine the wear across different treads. Each step may have a distinct normal force distribution or a slightly different sliding distance. In a more refined model, these step-by-step differences can be factored in by using an appropriate average value of F and d for each step or by performing a separate calculation for each tread.

This approach provides a direct way to estimate the average usage rate of the stairs from the total worn volume. If λ derived in this manner matches historical records or other indications of occupancy, then the wear patterns can be viewed as consistent with the known or hypothesized frequency of use.

### 3.3.3 Flow patterns related to the use of stairs

In this subsection, we propose a simplified micro-scale model to determine whether ascending or descending traffic predominates. The following assumptions are made:

- The force and wear per traversal are identical for both ascending and descending traffic.
- Each stair tread is divided into two distinct micro-regions: an edge region, predominantly worn by ascending traffic due to the forward push-off, and a center region, which experiences wear from both ascending and descending traffic.

Under these assumptions, comparing the wear volumes in the edge and center regions allows us to estimate the fraction of ascending versus descending traversals. An illustration of this approach is shown in Figure 6, where Region 1 represents the center region, and Region 2 represents the edge region.

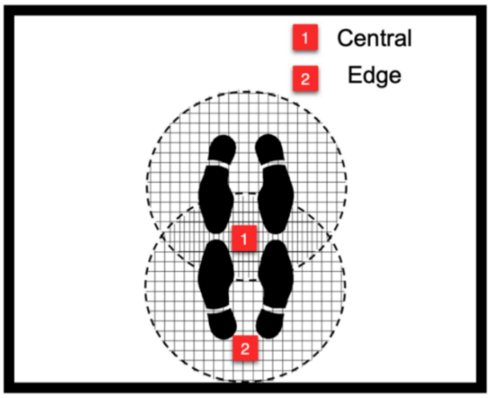


Figure 6: Illustration of Ascending and Descending Traffic

Let $E$ denote the number of micro-elements or the area (in $cm^2$ ) of the edge region, and C denote the number of micro-elements in the center region. Let $v_\varepsilon$ and $v_C$ represent the wear per traversal for edge and center elements, respectively. The total number of stair traversals is denoted by $Q$, with an unknown fraction p representing the proportion of ascending trips. Thus, we can express the number of ascending and descending traversals as:

$$\text{ascending traversals} = p*Q, \qquad \text{descending traversals} = (1-p)Q \tag{9}$$

Since the edge region is worn only by ascending traffic, each edge element experiences wear during pQ traversals. In contrast, the center region is worn by all traffic, i.e., Q traversals.

The wear models for the edge and center regions are as follows:

**Edge Wear**:

$$V_{\mathcal{E}} = \sum_{i\in\mathcal{E}}(pQ \times v_{\mathcal{E}}) \approx \mathcal{E}(pQ)v_{\mathcal{E}} \tag{10}$$

Assuming each edge element has approximately the same wear contribution $V_{\mathcal{E}}$ per traversal.

**Center Wear**:

$$V_{\mathcal{C}} = \sum_{i\in\mathcal{C}}(Q \times v_{\mathcal{C}}) = CQV_C \tag{11}$$

Since the center region is used by both ascending and descending traffic at equal rates. From the equation for center wear, we can solve for Q as:

$$Q = \frac{V_{\mathcal{C}}^{(\text{obs})}}{Cv_C} \tag{12}$$

Substituting this value of Q into the edge wear equation, we can solve for p:

$$V_{\mathcal{E}}^{(\text{obs})} = \mathcal{E}\left(p \times \frac{V_{\mathcal{C}}^{(obs)}}{\mathcal{C}v_{\mathcal{C}}}\right)v_{\mathcal{E}} \tag{13}$$

Which simplifies to:

$$p = \frac{V_{\mathcal{E}}^{(\text{oba})}}{Ev_{\mathcal{E}}} \times \frac{Cv_C}{V_C^{(\text{oba})}} \tag{14}$$

If p > 0.5, it implies that more than half of the traversals were ascending, indicating that the stair was predominantly used for ascending. Conversely, if p < 0.5, it suggests that descending traffic exceeds ascending traffic. A balanced usage is implied when $p \approx 0.5$.

This method, where edge wear is attributed exclusively to ascending traffic and center wear is attributed to both directions, provides an approximate estimate of the ratio of ascending to descending usage. While some real-world variations may exist (e.g., if some descending travelers also use the edge region), this basic comparison between edge and center wear volumes offers a reliable micro-scale assessment of directional traffic preferences.

### 3.3.4 the average number of people using the stairs at the same time

Building upon the previous model, this section aims to determine the average number of people using the stairs at the same time. Unlike the earlier approach, which assumes a uniform distribution of the contact force, we now take a more continuous approach, allowing for variability in force application across different regions of the stair tread. Specifically, we consider scenarios where either one or two people traverse the stairs together.

To facilitate a detailed analysis, each stair tread is divided into six discrete footprints, labeled from left to right as j = 1, 2,..., 6. Each footprint j consists of multiple micro-elements, and the wear volume of each element within footprint j is denoted as $V_{ij}$, where i indexes the individual micro-elements ($V_{ij} = V_{i1}, V_{i2},..., V_{i6}$). Figure 7 illustrates how a footprint might appear.

When a single person uses the stairs, wear occurs primarily in the central footprints (regions 3 and 4). In contrast, when two people use the stairs simultaneously, the person on the left wears the left footprints (regions 1 and 2), and the person on the right wears the right footprints (regions 5 and 6). This results in different wear patterns:

Single User Traversal: Wear occurs in regions 3 and 4, following a normal distribution centered at $\mu_j$ along the x-axis.

Double User Traversal: Wear occurs in regions 1 and 2 (left side) and regions 5 and 6 (right side), each following their respective normal distributions centered at $\mu_1$ and $\mu_6$.

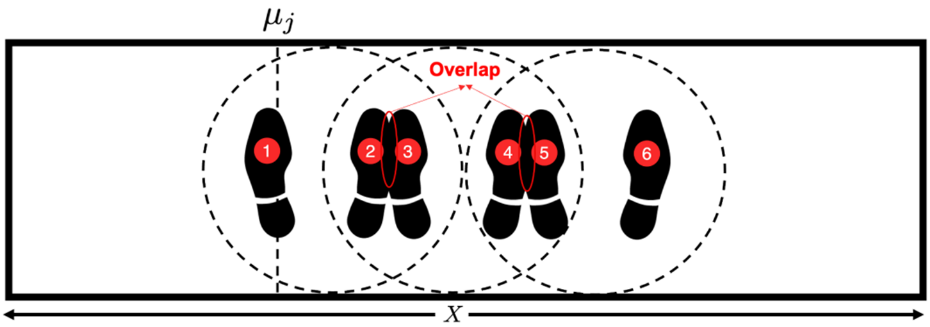


Figure 7: Footprint Distribution

We model the force exerted on each footprint as a normal distribution along the x-axis. Let $f_j$ represent the force applied to footprint j, where $f_j$ =F/2 for double traversals, assuming the total force F is evenly distributed between the two users. The force distribution within each footprint is modeled as:

$$f_j(x) = \frac{F}{2} \cdot \exp\left(-\frac{(x-\mu_j)^2}{2\sigma^2}\right) \tag{15}$$

where x is the position along the x-axis within the footprint. μj is the center position of footprint j. σ controls the spread of the force distribution.

The wear volume $V_{ij}$ for each micro-element within footprint $j$ is calculated using Archard's Wear Law:

$$V_{ij} = \frac{K \cdot f_j(x) \cdot 1}{H} \tag{16}$$

Observed Wear Volumes: $V_{i1}^{(obs)}$ and $V_{i6}^{(obs)}$: Wear volumes in the leftmost and rightmost footprints, primarily attributed to double traversals. $V_{i2}^{(obs)}$ and $V_{i5}^{(obs)}$: Wear volumes in the adjacent central footprints, resulting from both single and double traversals. $V_{i3}^{(obs)}$ and $V_{i4}^{(obs)}$: Wear volumes in the innermost central footprints, predominantly from single traversals but also influenced by overlapping wear from double traversals.

**Model Equations**: For edge footprints (j = 1, 6):

$$V_{i1}^{(obs)} = P_{double} \cdot V_{double}, \qquad V_{i6}^{(obs)} = P_{double} \cdot V_{double} \tag{17}$$

Sum of the wear contributions from both users, resulting in higher wear volumes compared to the edge regions (1 and 6). Mathematically, for overlapping footprints:

$$V_{i2} = V_{i3} = V_{\text{single}} + V_{double} \tag{18}$$

Where $V_{\text{single}}$ is the wear volume from a single traversal, and $V_{double}$ is the additional wear from simultaneous traversals.

$$f_j(x) = \frac{F}{2} \cdot \exp\left(-\frac{(x-\mu_j)^2}{2\sigma^2}\right) \tag{19}$$

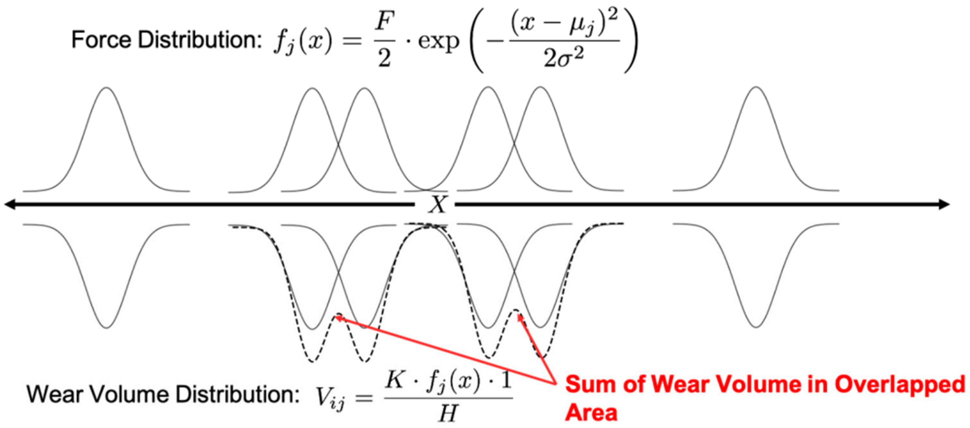


Figure 8: Sum of Wear Volumes in Overlapping Areas

Let $P_{double}$ denote the probability that two people use the stairs simultaneously. To infer $P_{double}$, we compare the observed wear volumes across different footprints:

**Observed Wear Volumes**:

$\boldsymbol{V_{i1}^{(obs)}}$ **and** $\boldsymbol{V_{i6}^{(obs)}}$: Wear volumes in the leftmost and rightmost footprints, primarily attributed to double traversals.

$\boldsymbol{V_{i2}^{(obs)}}$ **and** $\boldsymbol{V_{i5}^{(obs)}}$: Wear volumes in the adjacent central footprints, resulting from both single and double traversals.

$\boldsymbol{V_{i3}^{(obs)}}$ **and** $\boldsymbol{V_{i4}^{(obs)}}$: Wear volumes in the innermost central footprints, predominantly from single traversals but also influenced by overlapping wear from double traversals.

**Model Equations**: For edge footprints (*j* = 1, 6):

$$V_{i1}^{(obs)} = P_{double} \cdot V_{double}, \qquad V_{i6}^{(obs)} = P_{double} \cdot V_{double} \tag{20}$$

For adjacent central footprints (j = 2, 5):

$$V_{i2}^{(obs)} = V_{i5}^{(obs)} = (1 - P_{double}) \cdot V_{\text{single}} + P_{double} \cdot V_{double} \tag{21}$$

For innermost central footprints (j = 3, 4):

$$V_{i3}^{(obs)} = V_{i4}^{(obs)} = (1 - P_{double}) \cdot V_{\text{single}} \tag{22}$$

Solving for $P_{double}$ : From the edge footprints

$$p_{\text{double}} = \frac{V_{i1}^{(obs)}}{V_{\text{double}}} = \frac{V_{i6}^{(obs)}}{V_{\text{double}}} \tag{23}$$

From the adjacent central footprints:

$$p_{\text{double}} = \frac{V_{\text{i2}}^{(\text{obs})} - (1 - p_{\text{double}}) \cdot V_{\text{single}}}{V_{\text{double}}} = \frac{V_{\text{i5}}^{(\text{oba})} - (1 - p_{\text{double}}) \cdot V_{\text{single}}}{V_{\text{double}}} \tag{24}$$

To further refine the model, we consider a continuous distribution of wear across the x-axis. Instead of discrete footprints, the wear volume varies smoothly, modeled by Gaussian distributions centered at different positions:

**Single Traversal**: Wear is concentrated around the center ($j$ = 3, 4) following a normal distribution.

**Double Traversal**: Wear extends toward the edges ($j$ = 1, 2, 5, 6) with overlapping normal distributions from both users.

The wear volume for each micro-element $V_{ij}$ is computed as:

$$V_{ij}^{(obs)} = \mathrm{n} \cdot V_{ij}^{(single)} + \mathrm{m} \cdot V_{ij}^{(double)} \tag{25}$$

where $n$ is the number of single traversals. m is the number of double traversals. $V_{ij}^{(single)}$ and $V_{ij}^{(double)}$ are the wear volumes from single and double traversals, respectively, calculated using the normal distribution profiles.

By fitting the observed wear volumes to this continuous model, we can estimate the probability $p_{\mathrm{double}} = \frac{n}{n+m}$, indicating the likelihood of simultaneous stair usage.This method remains applicable even when there are more users involved.

Figure 9: Illustration of Multiple Footprints

Table 2: Wear Volume Data

| Variable | Vi1(obs) | Vi2(obs) | Vi3(obs) | Vi4(obs) | Vi5(obs) | Vi6(obs) | Vsingle | Vdouble |
|---|---|---|---|---|---|---|---|---|
| Value (cm3 ) | 15 | 30 | 50 | 50 | 30 | 15 | 10 | 15 |

Consider the following observed wear volumes and estimated parameters:

**From the edge footprints**:

$$\mathrm{p_{double}} = \frac{\mathrm{V_{i1}^{(obs)}}}{\mathrm{V_{double}}} = \frac{15}{15} = 1 \tag{26}$$

**For the central footprints**:

$$\mathrm{n} = \frac{\mathrm{V_{i3}^{(obs)}}}{\mathrm{V_{single}}} = \frac{50}{10} = 5 \tag{27}$$

$$n = \frac{V_{i4}^{(obs)}}{V_{\mathrm{single}}} = \frac{50}{10} = 5 \tag{28}$$

Thus, the total number of traversals Q = n + m = 5 + 1 = 6, and the probability of double usage is:

$$p_{\mathrm{double}} = \frac{1}{6} \approx 0.167 \tag{29}$$

This indicates that approximately 16.7% of stair usage involved two individuals simultaneously using the stairs.

### 3.4 Extended Inferences and Historical Consistency

#### 3.4.1 the Age of the Stairwell

In this section, we estimate the age of the stairwell, which is a crucial factor in understanding the wear patterns and their correlation with usage. Since the true age of the stairwell is often unknown, we use a probabilistic approach to estimate it. The stairwell age is treated as a random variable T, representing the unknown true age. We assume that the estimate of the stairwell's age, $\hat{T}$, follows a normal distribution:

$$\hat{T} \sim N(\mu, \sigma^2) \tag{30}$$

We plot the probability density function (PDF) of the normal distribution for $\hat{T}$. As shown in Figure 11, the plot illustrates the distribution with a mean of μ = 50 years and a standard deviation σ= 10 years.

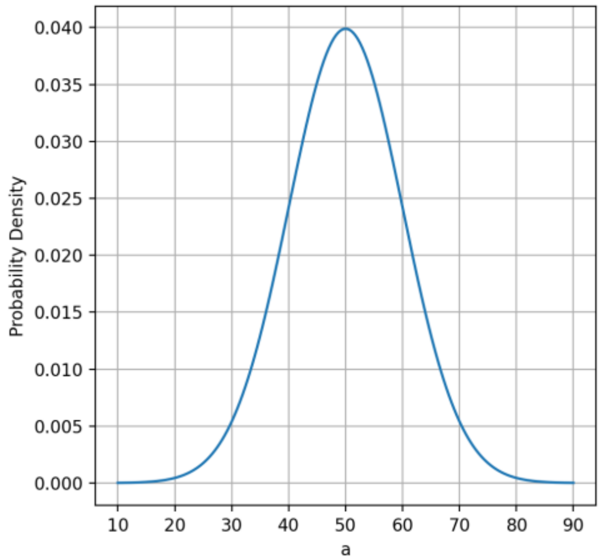


Figure10: Random distribution of a, with mean μa = 50 and standard deviation σa = 10.

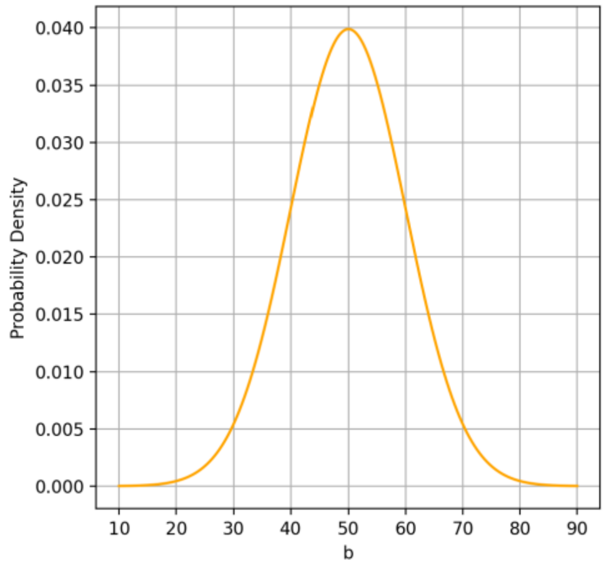


Figure 11: Random distribution of b, with mean μb = 50 and standard deviation σb = 10.

In addition, since the stairwell's age estimation is often influenced by multiple factors, we may also model the relationship between two variables, say a and b, each following a normal distribution. The product of these two variables, a • b, will also be a random variable. This is an important consideration because the combination of these factors will also influence the age estimate. This distribution can be visualized by plotting the individual distributions of a and b, as well as the joint distribution of a • b.

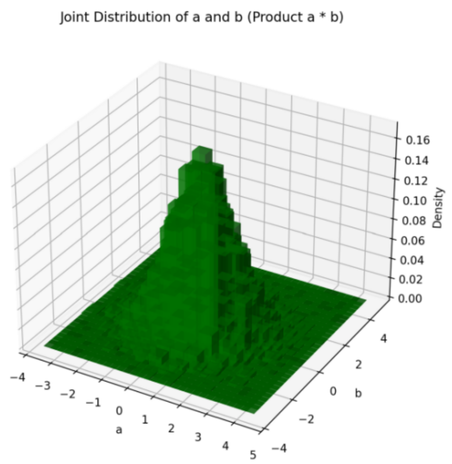


Figure 12: Joint distribution of a • b

We can estimate the age T of the stair surface assuming a constant annual wear rate:

$$T = \frac{V_{\text{total}}}{\text{Annual Wear Rate}} \tag{31}$$

The annual wear rate is typically determined from historical data or experimental measurements, and V is the measured wear volume.

### 3.4.2 Material Heterogenity

The type and material of the material can be indicated by its wear rate.

The wear volume $v$ is determined by material properties, applied load, and sliding distance:

$$v = \frac{K \cdot F \cdot S}{H} \tag{32}$$

where K is the wear coefficient, a dimensionless constant that reflects the material's susceptibility to wear; F is the normal force (unit: N), which represents the body weight or contact pressure; S is the total sliding distance (unit: cm), which is often equivalent to the single-step sliding distance s in experiments; and H is the material hardness (unit: $N/mm^2$ ).

We then define the wear rate *W*, which represents the wear volume per unit contact area, calculated as:

$$W = \frac{v}{N_{\text{total}}} \tag{33}$$

Substituting *V* and *N* into this equation, we obtain:

$$W = \frac{\frac{K \cdot F \cdot S}{H}}{s \cdot L} = \frac{K \cdot F}{H \cdot L} \tag{34}$$

Assuming that K, H, and L are constants, the wear rate *W* is directly proportional to the applied load F :

$$W = \left(\frac{K}{H \cdot L}\right) \cdot F \tag{35}$$

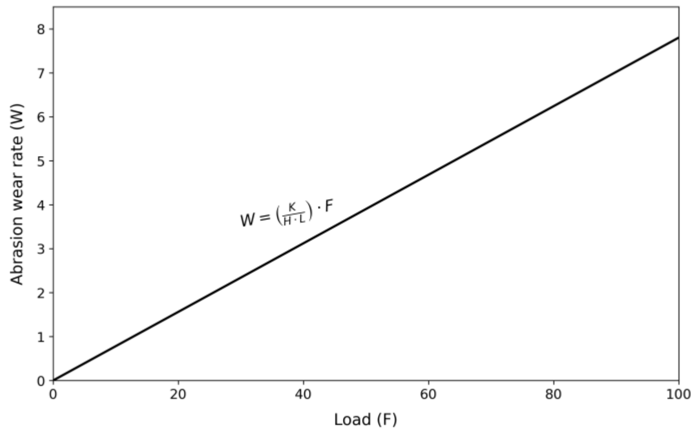


Figure 13: Abrasion wear rate versus applied load

In this relationship, the slope $\frac{K}{H \cdot L}$ depends on the material properties (i.e., K/H) and the foot width *L*. Further analysis leads to the following conclusions:

- Firstly, different types of stone exhibit distinct slopes due to variations in K/H. This can be observed in the experimental results.
- Meanwhile, the wear rate of the material is highly correlated with its physical natural quantities (such as density, natural unit weight, saturated unit weight, dry unit weight, porosity, etc.).

By reversing the calculation based on the wear rate for multiple physical quantities, and comparing them with the actual physical values of the stone materials from nearby quarries obtained through experiments, it is possible to determine the material and origin of the stone materials.

## 4. Conclusion

This study successfully proposes a hybrid framework that integrates Archard's wear law with Convolutional Neural Networks (CNNs) to analyze staircase wear patterns and decode historical usage dynamics. By using non-destructive imaging techniques and precise mechanical model- ing, our framework provides actionable insights into usage frequency, directional preferences, group sizes, stair age, and material provenance. Additionally, this study advanced material provenance identification by establishing correlations between wear rates and physical material properties, enabling the tracing of the staircase's material origins. This approach not only enhances the accuracy of wear analysis but also offers new tools for archaeological research and architectural heritage conservation.

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