

Mechanical and Motion Analysis of Volleyball

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Abstract. This paper deals with the mechanics and laws of the game of volleyball, such as the in-depth examination of the ball's flight in different conditions, the construction of a serving process mathematical model, and the examination of the most significant factors affecting the serving performance. Based on the theoretical analysis, modeling, and interpretation of the results, the paper explains the laws of the game of volleyball in the absence and presence of air resistance and offers theoretical support for the optimization of the serving technique by players and coaches. The results show that the accurate regulation of the takeoff angle and the initial serving speed will primarily contribute to serving success, and the input of air resistance will make the game of volleyball more realistic and have specific implications for practical training and competition.

Keywords: Volleyball; Mechanical analysis; Motion trajectory; Serving model; Air resistance.

1. Introduction

As a very popular sport, volleyball amazes millions of players and fans with its special competitive charm. For competitions in the game of volleyball, the precise control of the ball's flight and touch-down location is a key decider of the game's result [1]. Whether it's a booming serve that scores instantly or a beautiful pass that creates the attacking opportunity, the accomplishment relies to a very high degree on the player's accurate anticipation of the ball's flight.

A detailed examination of the mechanics and motion of volleyball is important in a variety of ways. For players, a knowledge of the mechanics of the sport of volleyball will enable them to optimize technical skills, make serving, passing, and hitting more precise and consistent, and ultimately lead to enhanced sports performance [2][3]. For instance, with knowledge of ideal serve velocity and serve angle, players can make serves that will be more dangerous and make it even more difficult for the opposing team to return the ball. For coaches, the mechanics of the sport offer the theoretical basis on which scientific training programs can be built [4]. With knowledge of the mechanics, coaches will be able to give more descriptive instructions, make better use of training time, and make the team more competitive overall.

Further, research in the mechanics of volleyball can also facilitate the design of sports equipment and the optimization of playing facilities. By studying the flight characteristics of volleyballs, designers can refine ball construction to better conform to aerodynamic principles and improve flight performance [5][6]. Likewise, mechanical analysis of court surface materials and elasticity can support the creation of playing environments that enhance athlete performance.

In recent years, significant progress has been made in the field of mechanical and motion analysis of volleyball. In terms of kinematics, Bernardo et al. conducted in-depth studies on projectile motion under air resistance [7]. By solving the relevant equations, they successfully predicted the motion trajectory of objects [8]. This research provides an important theoretical foundation for analyzing volleyball motion in match environments. The mechanical and kinematic analysis of volleyball continues to evolve, with researchers employing experimental methods and numerical simulations to investigate the forces acting on the ball during flight, changes in velocity, rotational effects, and more. Some studies have focused on the motion characteristics of volleyball under different hitting techniques, analyzing how force transmission at the moment of impact and the ball's initial state influence its subsequent trajectory [9].

However, there are still gaps in the current research. Although numerous studies have been conducted in the critical phase of serving, comprehensive consideration of the complex factors involved in the serving process remains insufficient. For instance, factors such as wind speed, direction during a serve, and player motion variations have not been fully studied in real-game scenarios. The interaction mechanisms between these factors also warrant further exploration.

This study focuses on the mechanics and motion characteristics of volleyball. First, under idealized conditions and assuming no air resistance, the study derives the equations of motion in both the horizontal and vertical directions based on classical projectile motion theory. It constructs a mathematical model of the serving process. Based on this, it discusses the appropriate ranges of serving speed and angle required for the ball to pass over the net and remain in bounds, thereby establishing a framework for theoretical analysis. Second, to better reflect actual match conditions, the study incorporates the influence of air resistance and formulates motion equations accordingly. It then analyzes how air resistance affects the volleyball's trajectory. Through numerical simulations, the study examines the trajectory variations of the ball under different initial speeds and launch angles. Finally, a systematic analysis of the simulation results identifies reasonable serving speed and angle ranges under various conditions. This research not only provides concrete reference data to improve serving success rates but also offers theoretical support for volleyball training and instructional practice.

2. Theory of projectile motion

Projectile motion refers to the motion of an object that is projected into the air and is subject only to the force of gravity and, in some cases, air resistance [10]. The motion of the object can be broken down into two components: horizontal and vertical. These components act independently of each other but occur simultaneously, giving rise to a parabolic trajectory.

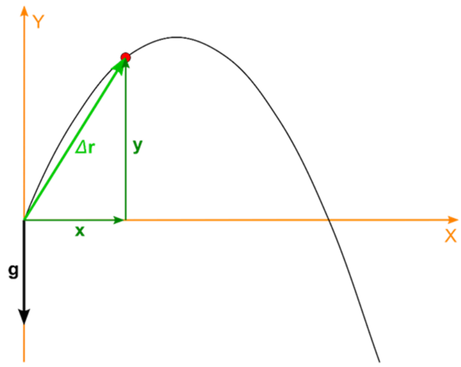

Fig.1 Diagram of Oblique Projectile Motion

2.1 Without air resistance

In volleyball, projectile motion is one of the fundamental forms of movement for the ball. When we assume that the air resistance is zero, the only force acting on the ball during its motion is gravity [11]. This simplifies the analysis of its trajectory, allowing us to focus solely on the gravitational force and its effect on the ball's motion. Under these conditions, the ball's acceleration is expressed as:

$$a_x = 0, a_y = -g \tag{1}$$

where g s the acceleration due to gravity, approximately $9.8ms^{-2}$. This means that in the horizontal direction, there is no acceleration (since $a_x = 0$), and the motion proceeds with a constant horizontal velocity. In the vertical direction, the acceleration is constant and directed downwards due to gravity.

The velocity of the ball can be decomposed into two components—horizontal (v_x) and vertical (v_y)—based on the initial velocity u and the angle of projection θ.

Horizontal Component (x-direction): The horizontal velocity remains constant throughout the flight of the ball since no external forces are acting in the x-direction (neglecting air resistance). The horizontal displacement x over time t is given by:

$$x = u \cdot \cos(\theta) \ \cdot t \tag{2}$$

where $u \cdot \cos\theta$ represents the horizontal component of the initial velocity.

Vertical Component (y-direction): Vertical movement is affected by gravity, with the ball slowing down as it travels upwards and speeding up as it comes back down. The vertical displacement y and vertical velocity v_y at any time t are given by:

$$y = u \cdot \sin(\theta) - \frac{1}{2} g \cdot t^2 \tag{3}$$

$$v_y = u \cdot \sin(\theta) - g \cdot t \tag{4}$$

where $u \cdot \sin(\theta)$ is the initial vertical velocity, and g is the acceleration due to gravity.

In a serve in a game of volleyball, the aim is not only to get the ball over the net but also to the court within the lines. To accomplish that purpose, we will analyze two conditions of significance:

C1: The ball must pass over the net.

C2: The ball must land within the court's boundaries (without going out of bounds).

In order to meet these requirements, we analyze the serve using the equations of projectile motion to the ball's vertical and horizontal displacement and velocity.

2.1.1 Ensuring the Ball Passes Over the Net

In order to ensure the ball will clear the net, we must determine the height of the ball when it crosses the net at some horizontal position from the server. To achieve that, we use the given kinematic equations:

Horizontal velocity: The horizontal component of velocity remains the same throughout the motion and equals:

$$Vx = v_0 \cos(\theta) \tag{5}$$

where v_0 is the initial velocity of the ball, and $\cos(\theta)$ is the angle at which the ball is serving.

Time to travel a given horizontal distance: The time to reach the net by the ball, a distance from the server, is given by:

$$t = s \cdot v_x = s \cdot v_0 \cdot \cos(\theta) \tag{6}$$

Vertical position of the ball at time *t*: The ball's vertical displacement y at time t will be determined by the vertical velocity and the effect of gravity:

$$y = v_0 \cdot \sin(\theta) \cdot t - 1/2 \cdot gt^2 + h_0 \tag{7}$$

Where h_0 is the starting height of the serve (usually from the hand of the player), and g is the acceleration due to gravity (9.8 ms^{-2}).

To ensure the ball clears the net, the vertical displacement y must be greater than the height of the net, which is 2.24 meters:

$$y = v_0 \cdot \sin(\theta) \cdot t - 1/2 \cdot gt^2 + h_0 > 2.24 \tag{8}$$

This inequality ensures that the ball passes over the net.

2.1.2 Ensuring the Ball Stays Within the Court

After ensuring the ball clears the net, the next step is to make sure the ball lands within the court’s boundaries, which are 18 meters long. To do this, we analyze the motion until the ball hits the ground:

Vertical displacement when the ball hits the ground: When the ball hits the ground, its vertical displacement becomes zero. Therefore, we set y=0 in the vertical displacement equation:

$$v_0 \cdot \sin(\theta) \cdot t - 1/2 \cdot gt^2 + h_0 = 0 \quad (9)$$

This quadratic equation can be solved for t_0, the time at which the ball lands on the ground.

Horizontal distance traveled by the ball: Once we have t_0, the total time of flight, we can calculate the horizontal distance x that the ball has traveled by the time it lands:

$$x = V_x \cdot t_0 = v_0 \cdot \cos(\theta) \cdot t_0 \quad (10)$$

For the ball to remain in bounds, this horizontal distance must be less than or equal to 18 meters:

$$x = V_x \cdot t_0 = v_0 \cdot \cos(\theta) \cdot t_0 \leq 18 \quad (11)$$

If the calculated value exceeds 18 meters, the serve will go out of bounds.

By combining these two conditions—passing over the net and staying within the boundaries—we can derive an optimal range for the initial velocity v_0 and the angle of projection θ. This ensures that the volleyball serve is both successful (passes over the net) and legal (stays within bounds).

If we consider the volleyball court in 3D, the analysis becomes more complex, but it allows for a more complete understanding of the serve. In a real game, the serve can travel diagonally, making use of both the court's length and width.

2.2 With air resistance

In reality, volleyball does not move in a vacuum. Air resistance, which acts opposite to the direction of the ball's motion, influences its trajectory and reduces both the horizontal and vertical velocities over time [12]. Air resistance depends on factors such as the ball's speed, geometry, volume, and the density of air.

Horizontal Motion with Air Resistance: Without air resistance, the horizontal velocity (how fast the ball moves forward) stays constant because no force is acting on it in that direction. But when air resistance is present, it slows the ball down. This means the volleyball's horizontal velocity will gradually decrease over time. As a result, the ball won't travel as far as it would in a situation without air resistance.

Vertical Motion with Air Resistance: In the vertical direction, air resistance works against the ball's motion as well. Gravity is already pulling the ball downward, but air resistance adds an additional force that resists the ball's motion in both directions:

- When the ball is going up, air resistance slows it down faster than gravity alone, so the ball won't go as high.
- When the ball is coming down, air resistance reduces how quickly the ball accelerates. Gravity would normally make it fall faster and faster, but air resistance slows that acceleration, so the ball takes longer to hit the ground than it would without air resistance.

Forces acting on the ball.

Gravity $F_g = mg$, where m is the mass of the volleyball and g is the acceleration due to gravity.

Air resistance $F_{drag} = \frac{1}{2} C_d \rho A v^2$, where C_n is the drag coefficient (related to the shape of the volleyball), ρ is the air density, A is the projected area of the volleyball in the direction of motion, and v is the velocity of the volleyball.

Equations of Motion with Air Resistance: Air resistance opposes motion, so the acceleration components are:

Horizontal (x-direction): $\frac{dv_x}{dt} = -\frac{C_d \rho A v v_x}{2m}$

Vertical (y-direction): $\frac{dv_y}{dt} = -g - \frac{C_d \rho A v v_y}{2m}$

These equations describe the changes in the volleyball's motion under air resistance; however, they typically require numerical methods (such as numerical integration) to solve due to their complexity.

2.3 Properties of the trajectory

Whether under conditions with or without air resistance, the motion trajectory of a volleyball exhibits certain unique characteristics. In the absence of air resistance, the trajectory of the volleyball is a standard parabola, determined by the initial velocity and launch angle. As the initial velocity increases, both the flight distance and height of the volleyball increase. The launch angle influences the direction of flight and the curvature of the trajectory. When the launch angle is 45°, the horizontal flight distance reaches its maximum for a given initial velocity (ignoring court limitations).

With air resistance, however, the trajectory is no longer a standard parabola. Due to the effect of air resistance, the trajectory becomes distorted and flatter. During the ascent phase, the slope of the trajectory changes more rapidly, meaning the volleyball's upward velocity decreases faster. In the descent phase, the slope changes more gradually, and the volleyball's downward velocity increases more slowly. Moreover, air resistance significantly shortens the flight distance of the volleyball, especially at higher speeds where its influence becomes more pronounced.

3. Mathematical model of volleyball serving

The court model is shown in Fig. 2. During the serving process, in order for the ball to successfully clear the net and remain within bounds, a mathematical model can be established based on the previous theoretical analysis.

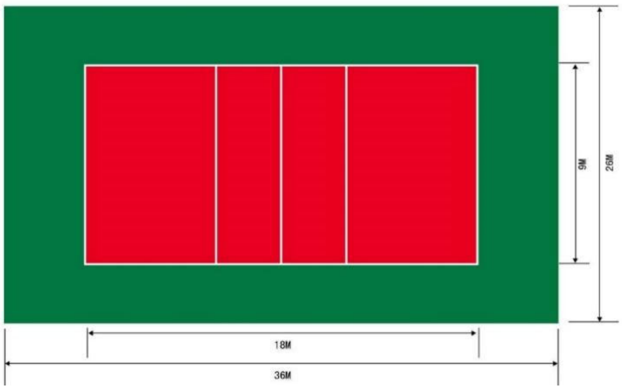

Fig. 2 Schematic Diagram of the Court Model

Assume the coordinates of the serving point are $(0, h_0)$, the net is located at a horizontal distance s from the serving point with a height of h_{net}, and the boundary of the court is at a horizontal distance L from the serving point (L = 18 meters).

Net clearance condition: According to the previously derived equations, the vertical displacement y of the ball at the horizontal distance *s* must satisfy:

$$y = u_0 \sin(\theta) \cdot \frac{s}{u_0 \cos(\theta)} - \frac{1}{2} g \cdot \left(\frac{s}{u_0 \cos(\theta)}\right)^2 + h_0 \tag{12}$$

Rearranging yields $s \tan(\theta) - \frac{gs^2}{2u_0^2 \cos^2(\theta)} + h_0 > h_{net}$

Out-of-bounds condition: From the vertical displacement equation with respect to $v_0 \sin(\theta) \cdot t - \frac{1}{2} g t^2 + h_0 = 0$, the landing time t_0 of the ball can be obtained using the quadratic formula: $t_0 = \frac{u_0 \sin(\theta) \pm \sqrt{u_0^2 + \sin^2(\theta) - 2gh_0}}{g}$.

Take the positive solution $t_0 = \frac{u_0 \sin(\theta) + \sqrt{u_0^2 + \sin^2(\theta) - 2gh_0}}{g}$ (since time cannot be negative).

The horizontal distance when the ball lands is given by: $x = v_0 \cos(\theta) \cdot t_0$. To ensure the ball does not go out of bounds, the following condition must be satisfied:

$$\frac{u_0 \cos(\theta)\left(u_0 \sin(\theta) + \sqrt{u_0^2 + \sin^2(\theta) - 2gh_0}\right)}{g} \le L \tag{13}$$

Combining these two conditions can derive a system of inequalities involving the initial velocity v_0 and the launch angle θ. Solving this system allows us to determine the valid ranges of v_0 and θ that ensure a successful serve at different serving heights h_0.

4. Results and discussion

The effects of different initial velocities v_0 and launch angles θ on serving performance were investigated through numerical computation and simulation. The following are some representative calculation results:

Table 1 Calculation Results

Launch Angle	Horizontal Range (R)	Maximum Height (H)	Flight Time (T)	Motion Characteristics
90°	0.0 m	5.0 m	2.00 s	Vertically launched, no horizontal displacement, the longest flight time.
75°	5.0 m	4.7 m	1.93 s	High - parabolic trajectory, short range but long airtime.
60°	8.7 m	3.7 m	1.73 s	Balanced height and range, close to the theoretical optimal angle.
45°	10.0 m	2.5 m	1.41 s	Maximum range (theoretical value without air resistance).
30°	8.7m	1.2 m	1.00 s	Low - parabolic trajectory, the same range as 60° but shorter time.
15°	5.0 m	0.3 m	0.52 s	Launched nearly horizontally, lands quickly.

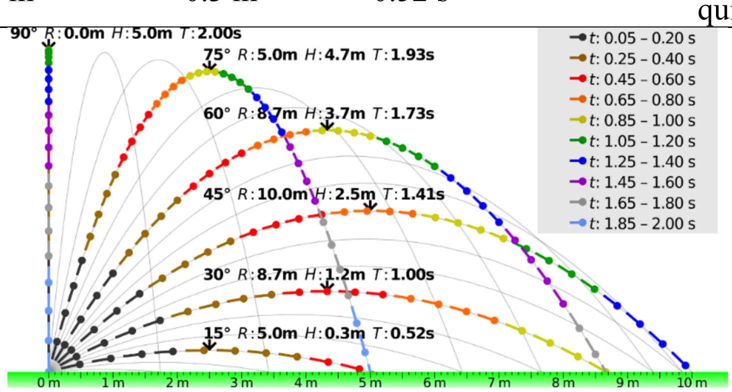

Fig. 3 Volleyball trajectories at different launch angles

From the results, it can be observed that the launch angle has a significant impact on the trajectory of the volleyball. When the launch angle is 90°, the ball moves vertically upward, resulting in a horizontal range of zero. As the launch angle decreases, the horizontal range first increases and then decreases, reaching its maximum at approximately 45° (under ideal conditions without air resistance and ignoring court boundaries). However, in actual volleyball serves, due to the need to clear the net and remain within bounds, the optimal launch angle is not exactly 45°.

Regarding the net-clearance condition, when the launch angle is too small, the ball's initial vertical velocity may be insufficient to pass over the net. Conversely, with a very large launch angle, although the ball can easily clear the net, it may descend quickly and fail to travel a sufficient horizontal distance, increasing the risk of going out of bounds. For instance, at a 15° launch angle, the ball's maximum height is only 0.3 meters. The ball may not clear the net if the serving height is not high enough.

As for staying in bounds, the launch angle and initial velocity must be properly coordinated. A higher initial velocity allows the ball to travel farther, but if the launch angle is not suitable, the ball can still go out of bounds. For example, at a 60° launch angle, the ball's horizontal range is 8.7 meters. If the initial velocity is too high, the ball could exceed the 18-meter court boundary.

Considering both net-clearance and in-bounds requirements, for a typical serve height (e.g., 2–3 meters), an initial speed between 10–15 m/s and a launch angle between 30°–50° are more likely to ensure a successful serve.Of course, it's just an estimated figure; the competitor's personal technique and the race conditions, such as the wind direction and velocity, also contribute to the actual run time.

Also, the presence of air resistance complicates the serving motion. When the air resistance dampens the ball, the flight range and highest height of the ball will be considerably shorter at the same launching speed and angle. Therefore, in actual gameplay, the players must adjust the launching speed and angle in accordance with the environments, such as the presence of winds, to obtain the best serving performance.

5. Conclusion

This paper provides a thorough and complete analysis of the characteristics and dynamics of the motion of a volleyball in the absence and presence of air resistance and constructs a mathematical model of the serving process.

In the theoretical case without air resistance, the equations of motion in the vertical and horizontal directions were worked out in detail based on the fundamental principles of projectile motion. The significant conditions under which the ball clears the net and remains in bounds were provided. Theoretical serving velocity and serving angle range were determined from a mathematically developed model. With air resistance, the action of air resistance on the flight of the ball was described, and the equations of motion in relation to each other were determined. Although the equations of motion are more difficult to solve, they better describe the ball's motion for practical purposes.

By simulation and analysis of various launching speeds and angles, it was concluded that control of the above two parameters to a certain extent is needed to maximize the rate of successful serves. Based on practical demands and taking into account the constraints of the net and boundaries, it was recommended that the range of launching speed should be 10–15 m/s and the launching angle between 30°–50° to maximize the rate of a successful serve. In actual volleyball games, however, things were more complex, and the implementation of serves is also dependent on such factors as individual technique differences between the players, court conditions (wind velocity, wind direction, and surface), and psychological factors. For example, master players were able to adjust their serve better based on the court conditions, whereas amateur players were able to judge and adapt to these factors more with hardship. The wind speed and direction also have a very important effect on the ball flight; in the tailwind condition, it may be preferable to reduce the launching speed and angle, and in the headwind, to increase them in order to maintain the quality of serves.

The results of the research provide theoretical advice of the highest importance to coaches and players of the game of volleyball. Players are able to use the results to inform the utilization of rational serving methods and conduct more targeted training to increase service quality. Coaches can also apply the principles of mechanics to training and game planning to make training scientific and efficient. Future research in the short to medium term could include the addition of more factors of simulation to the game of volleyball, e.g., the effect of spin imparted to the ball on the ball flight and the distribution of forces and the differences between the mechanics of volleyballs of differing substances. New motion capture and sensor-based measuring technologies could also be applied to obtain more precise data on the motion of the game of volleyball to permit better validation and improvement of theoretical models and to inspire further innovation in the techniques of the game and in training courses.

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