

Interdisciplinary Study on the Wear Mechanism and Service Mode of Stairs Based on Finite Element Fatigue Model

Xichen Zhang^{1*}, Xiangyu Guo^{2,a}

¹Dundee International Institute, Central South University, Changsha, China.

²School of Automation, Central South University, Changsha, China.

*7801220234@csu.edu.cn

^agoforit202209@163.com

Abstract. The wear trace of stone historical stairs can reflect key information such as its frequency of use, the distribution of people and the repair history, but traditional archaeological methods rely on qualitative observation, so it is difficult to quantify the dynamic fatigue process. The engineering model is difficult to adapt to the complex load environment of historical buildings because it simplifies the homogeneity of materials. In view of the above limitations, this study proposes a finite element fatigue wear model combining finite element analysis (FEA) and material fatigue theory and analyzes the wear mechanism and its historical correlation under periodic load by constructing a three-dimensional simulation model system. Taking typical stone materials (such as gneiss and serpentine) and wood materials (such as begonia and hard maple) as objects, combined with the S-N fatigue curve and wear law, the stress distribution and damage accumulation under different service frequencies are simulated, and multilateral fixed boundary conditions are introduced to approach the actual constraint state. The results show that the material properties play a decisive role in the wear process: stone is significantly better than wood because of its high hardness and shear modulus, and gneiss has the best wear resistance; There is a nonlinear positive correlation between the frequency of use and the wear rate, and the sensitivity of wood under high-frequency trampling is much higher than that of stone. Through long-term simulation and statistical verification (fitting test $R^2 > 0.98$), natural wear and artificial repair traces are successfully distinguished, such as judging the existence of renovation when the deviation of the fitting curve exceeds the threshold. The innovation of this study lies in the deep combination of FEA and material fatigue theory, which solves the limitation of traditional archaeological methods relying on qualitative analysis, provides a quantitative tool for the preventive protection of cultural relics, and restores the historical use mode through reverse derivation of wear characteristics. This study also realizes the cross-integration of material mechanics and archaeology and provides a new technical path for the micro-historical research and scientific protection of architectural heritage.

Keywords: Stair wear; Finite element analysis; Material fatigue; Interdisciplinary approach; Load simulation.

1. Introduction

The wear trace of stone stair tread can reflect the use frequency, crowd streamlines, and historical repair time. Traditional archaeological methods rely on qualitative observation and it is difficult to quantify material fatigue. However, the engineering wear model is mostly aimed at homogeneous materials and cannot reflect the complex load characteristics of historical buildings. Therefore, it has become a key issue for heritage protection to establish an evaluation method that can simultaneously quantify the fatigue-wear mechanism and reverse the historical use pattern.

The existing stair research mainly focuses on mechanical performance analysis. Rigo and Bagi [1] analyzed the mechanical behavior of cantilever stairs by discrete element simulation and pointed out that the connection mode of the tread end had a significant influence on the internal force distribution. Bangash [2] summarized the analysis method of stair structure and provided the standard comparison of design cases. Stepanov and Torskaya [3] found that the periodic interaction between the coating and rough surface was an important cause of material fatigue, and sliding speed and viscoelastic characteristics of the coating had significant effects on wear rate. These studies have limitations: first,

they mainly focus on the macro level, lacking the coupling analysis of fatigue and wear of micro-materials; second, the historical information behind the wear of stairs, such as the frequency of use and repair records, has not been fully excavated; third, the accuracy of the existing model is insufficient under complex boundary conditions and periodic loads.

This paper proposes a quantitative research model based on finite element analysis and material fatigue theory, aiming at accurately evaluating the wear state of stairs and mining the historical information behind them. This model simulates the fatigue damage and wear process of stairs under complex boundary conditions by constructing a finite element fatigue and wear model that integrates material parameters (such as Young's modulus and Poisson's ratio), load history (load frequency and load size) and archaeological data (renovation and pedestrian flow records). On this basis, the fatigue curve and wear law are introduced to analyze the wear laws of different types of materials (such as stone and wood) and their correlation with the frequency of use (measured by the average annual trampling times), thus providing a quantitative basis for the decision-making of cultural relics restoration. This interdisciplinary research method breaks through the limitations of traditional archaeological qualitative analysis and opens up a new path for the micro-history research of cultural relics.

2. Theoretical Method

In this paper, a finite element analysis is used to simulate the fatigue damage and wear of materials under periodic load, and the fatigue behavior of three-dimensional structures is studied. Considering the influence of the S-N curve, wear law, and fatigue on shear modulus, a finite element model is established to analyze the duration, age, renovation of stairs, and the influence of trampling with different frequencies.

The calculation of fatigue damage is usually based on the $S - N$ curve of material, which describes the relationship between stress amplitude and fatigue life and is expressed as:

$$\log(N) = S_N(\log(\sigma)) \quad (2.1.1)$$

Fatigue damage is:

$$f_d = \frac{1}{N} \quad (2.1.2)$$

According to Archard^[4] Wear law, the calculation formula of the wear rate \dot{v} is:

$$\dot{v} = K \cdot \frac{F \cdot \mu}{A} \quad (2.1.3)$$

Where F is the normal force, μ is the friction coefficient, A is the contact area and K is the wear coefficient.

The staircase itself is partially embedded and fixed, so the boundary conditions of fixed joints are introduced. For Dirichlet boundary conditions^[5] on the boundary Γ_D , supposing $u_D = 0$, then $u = 0$ and on the boundary Γ_D . For periodic boundary conditions^[6], assuming that there is a periodic relationship between displacement u and stress $\sigma(u)$ on the boundary Γ_p :

$$u_1 = u_2, \sigma(u_1) = \sigma(u_2) \quad (2.1.4)$$

Therefore, the complete finite element equation is as follows:

$$\int_0^\Omega \sigma(u): \varepsilon(u) d\Omega = \int_0^\Omega f \cdot v d\Omega + \int_0^{\Gamma_D} u \cdot v d\Gamma \quad (2.1.5)$$

The wear mechanism can be divided into fatigue wear and oxidation wear. Fatigue wear is caused by cyclic load, and its process begins with elastic-plastic deformation of the material surface under alternating stress. With the increase of stress cycles, micro-cracks are gradually formed on the surface microstructure of materials due to local stress concentration. These cracks usually extend along grain boundaries or phase boundaries and penetrate each other under subsequent loads. When the crack grows to the critical size, the surface layer of the material peels off and forms wear debris^[7]. The rate of fatigue wear is closely related to load amplitude, loading frequency, and fracture toughness of

materials. Higher load amplitude and loading frequency will significantly accelerate the initiation and propagation of cracks. The higher the fracture toughness of materials, the stronger the ability to resist crack propagation, thus slowing down the wear process.

Oxidative wear comes from the chemical reaction between the surface of the material and the environmental medium. In the presence of oxygen or corrosive gas, a brittle oxide film is formed on the surface of the material through an oxidation reaction. The difference in mechanical properties between the oxide film and the matrix material leads to its easy cracking under the action of external friction or shear stress. After the oxide film is broken, the exfoliated debris becomes abrasive particles, which further aggravates the surface wear [8]. The severity of oxidative wear is influenced by multiple factors: the increase of interfacial friction will directly promote the rupture of the oxide film. Although the increase in ambient temperature accelerates the reaction rate, it leads to a decrease in oxide film strength. The change of oxide layer thickness also regulates the wear mode. When the thin oxide layer is broken, fine abrasive particles are produced, while the rupture of the thick oxide layer is accompanied by the peeling of large-size debris, which leads to more serious abrasive wear. On this basis, the finite element fatigue wear model is used to analyze the performance degradation of the elastomer under long-term load, and the wear volume percentage is measured to judge whether it is consistent with the actual situation.

For the construction process of the model, the material properties and geometric parameters of the test are first set, as shown in Table 1. Adams [9] showed that the hard stone used in the model had relatively low fatigue wear under normal load, so the fatigue wear coefficient and fatigue index were set to 0.01 and 1.5, respectively. Because the stone surface is easy to oxidize, the oxidation wear process is stable, and it contributes greatly to the overall wear. The oxidation wear coefficient and oxidation index were set to 36 and 1 respectively. Based on Narimani et al. [10], the initial Poisson ratio is set to 0.26. Because of the hardness of stone, the Poisson's ratio is set to 0.3.

Table 1 Material Properties and Geometric Parameters

Parameter	Value	Unit
Fatigue wear coefficient	0.01	N/A
Fatigue index	15	N/A
Oxidation wear coefficient	36	N/A
Oxidation index	one	N/A
Length	1.2	m
Width	0.25	m
Initial shear modulus	35	GPa
Density	2.8	g/cm ³
Poisson's ratio	0.26	N/A
Poisson's ratio variation amplitude	0.3	N/A

The relationship between stress amplitude and fatigue life is established by dividing the discrete structure into grids and applying multilateral fixed boundary conditions and periodic loads. The displacement field and stress distribution are obtained by solving linear equations, and the overall fatigue life is predicted by combining the cumulative fatigue damage model. The finite element fatigue wear model simulates the elastic body subjected to periodic load, and the wear situation will be analyzed in the third part. On this basis, this paper predicts the service life and renovation of stairs, analyzes the long-term performance decline of materials by simulating periodic loads, predicts the volume percentage of structural wear, and compares it with the actual situation. Confidence analysis is introduced to simulate three material scenarios to evaluate the reliability of the model.

To detect whether the stairs are renovated, a discrete multi-point detection method is adopted, the fitting test is used to judge the confidence interval of the wear curve and a differential test is used to detect whether the annual attenuation exceeds the confidence interval. Through Monte Carlo random sampling, the data set based on the observation records of four archaeologists is selected to draw the relationship diagram between wear degree and time as shown in Figure 1. In the third section, the fitting test and differential test will be used for further analysis.

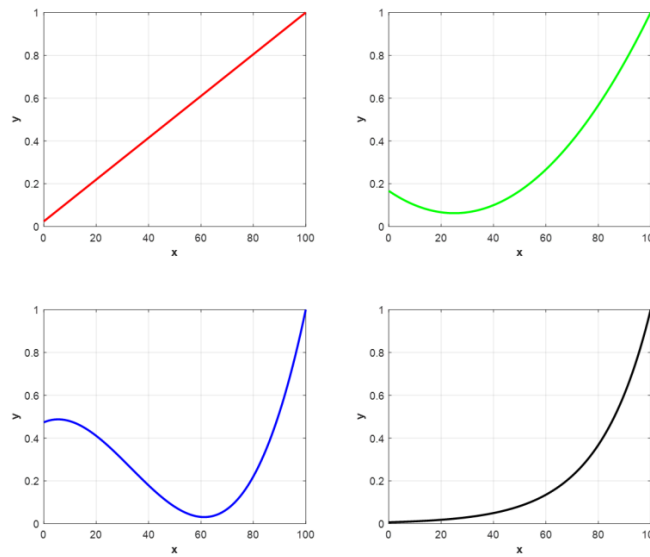


Figure 1 Wear Fitting Curve

After predicting the renovation situation, to determine the material source of stairs, gneiss and serpentine are selected as comparison stone samples in this section, while Begonia and hard maple are selected as comparison wood. The shear modulus and density of each material were measured and used as the key material parameters for analysis, as shown in Table 2.

Table 2 Material Parameters

Material type	Material name	Shear modulus (Gpa)	Density (g/cm3)
Stone material	Gneiss	29.5	2.84
Stone material	Serpentine	22.5	2.65
Wood	Begonia wood	0.85	0.65
Wood	Hard maple	1.1	0.685

This section deduces the fatigue curves of various materials through shear modulus, analyzes the influence of their time differences on fatigue strength, compares the wear rates of different materials, and determines their fatigue life, thus evaluating the differences between wood and stone.

Finally, the finite element fatigue wear model will explore the influence of using frequency on stair wear, and use the frequency analysis method to simulate the influence of high-frequency trampling on material fatigue:

$$G(t) = ae^{bt} \cdot f(t) \tag{2.1.6}$$

Where $G(t)$ is the damage rate, and $f(t)$ indicates the change of trampling frequency with time. In this section, the cyclic step-by-step model based on finite element, $f(t)$ will be replaced by $\sin(t)$, so the formula of the final fitting model is:

$$G(t) = ae^{bt} \cdot \sin(t) \tag{2.1.7}$$

Through the stair wear information recorded by the fitting model, the relevant data of stair use frequency can be deduced reversely. According to this model, the fluctuation diagram of wear rate is drawn, which will be analyzed in detail in the third part to get the relationship between wear rate and trampling frequency.

3. Practical Verification

The finite element fatigue and wear model simulates the elastic body under periodic load to study the change of material properties and solve the relationship between the age of stairs, maintenance, building materials comparison, and wear and trampling frequency. The model was loaded year by year for 400 years, and finally, the wear diagram was generated (see Figure 2).

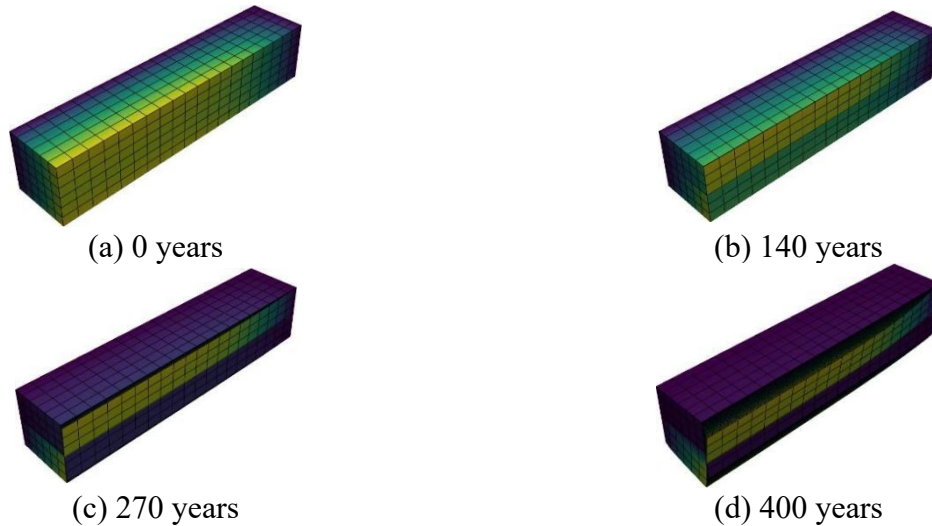


Figure 2 Wear Simulation of the Structure at Each Time Step

Figure 2 visually presents the stress amplitude distribution of the structure under different service life with color gradients. The lighter the color, the lower the stress level and the lighter the fatigue degree. The darker the color, the more serious the material corrosion and fatigue damage. At the initial stage of use (from 0 to 120 years), the overall color of the structure is light and evenly distributed, and the stress is at a low level and remains stable, which indicates that the material has good fatigue resistance and relatively light corrosion at this stage. However, with the increase in service time, the color of local areas began to darken, which indicated that stress gradually concentrated and fatigue damage began to sprout in these areas. By 270, the difference in color depth became significant, and the high-stress area further expanded. It is speculated that this is due to the propagation of micro-defects induced by long-term periodic load, which led to the accelerated development of fatigue damage and aggravated corrosion in local areas of materials. It is further speculated that in 400 years, the high-stress zone will spread further, and with the plastic deformation and cracking of materials, the structure will enter the accelerated period of fatigue damage, and the corrosion rate will also be significantly accelerated.

For predicting the service life of stairs, this model simulates three scenarios: the least pedestrians and the highest material strength, normal hardness and moderate number of pedestrians, and the most pedestrians but the most fragile materials (Figure 3). The fitting curve shows that the damage trend in these scenes is highly consistent with the actual data.

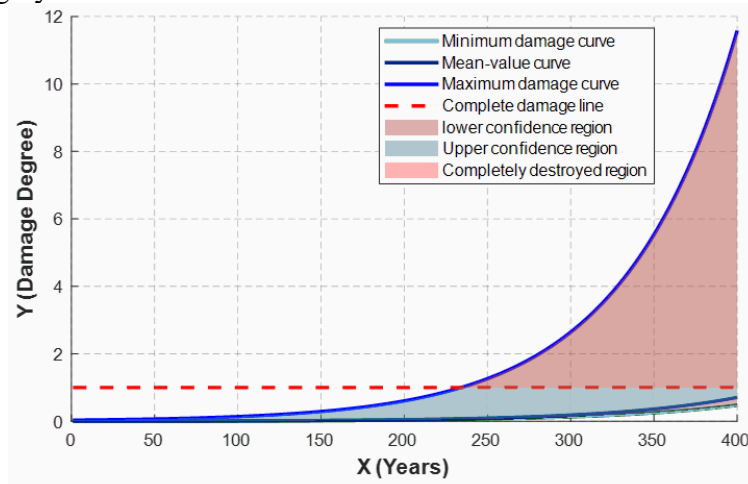


Figure 3 Exponential Fitting Curve of Stairs

Figure 3 shows that the damage of the hardest material stairs grows slowly, and the average curve reflects the overall damage aggravation, while the damage of the most vulnerable material rises sharply, exceeding the threshold of complete damage, and the completely damaged area expands significantly near 400 years. The actual wear value is within the confidence interval, which proves

that the prediction result of the model is reliable. Table 3 lists the results of the fitting test and difference test based on Figure 3 and Figure 2.

Table 3 Difference Test and Fitting Test Results

Dataset	R^2	p	Whether it passes the differential test or not.	Whether it passes the fitting test.
A	0.94587	0.001501	Yes	No
B	0.98306	0.022476	No	No
C	0.98104	0.001091	No	No
D	0.99452	0.208881	Yes	Yes

In the above table, R^2 is the determinant coefficient, which measures the fitting degree of the model to the data. The p value is used to test the significance of the influence of independent variables on dependent variables in the model. When $p > 0.05$, it is considered that the influence of year on wear is significant enough. The goodness of fit of group D is higher, and the sensitivity of the differential test is lower than that of the fitting test. The following figure shows the fitting results of fitting different observation records with exponential model $y = a \cdot e^{bx}$ in the fitting test:

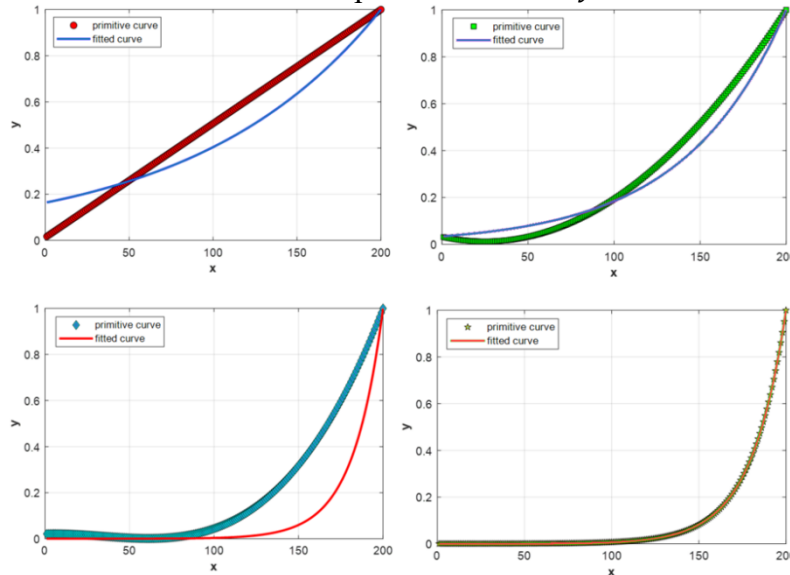


Figure 4 Fitting Test Results

Comparing the original curve and fitting curve in the observation records, it is found that the damage degree of D conforms to the fitting curve and has not undergone large-scale renovation, while the wear data of A, B, and C deviate significantly from the fitting curve, indicating that maintenance may be carried out. To determine the source of stair materials, gneiss, serpentine, begonia, and hard maple were selected in the second section, and their shear modulus and density were used to derive fatigue curves for comparison.

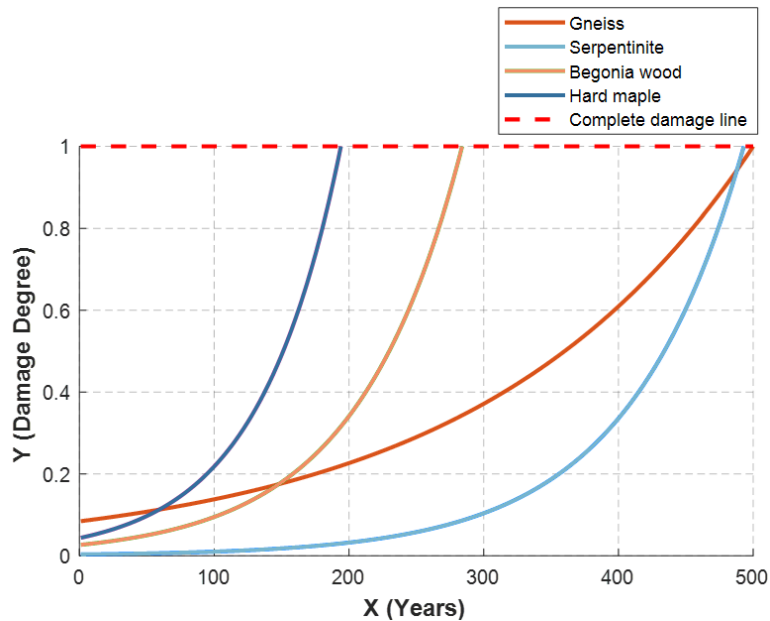


Figure 5 Wear Curves of Various Materials

As shown in Figure 5, the durability of stone is much higher than that of wood, gneiss in stone is better than serpentine, and hard maple in wood is better than begonia. Therefore, in actual construction, hard maple is used in parts that need higher strength and durability, such as stairs. Begonia wood, on the other hand, is built as a part with lower requirements for wear resistance, such as interior decoration. Based on the above cases, this model reveals the fatigue characteristics and wear laws of stair materials.

Finally, based on the finite element theory of formulas (2.1.6) and (2.1.7), this study discusses the relationship between high-frequency trampling frequency and material fatigue and draws the relationship diagram between trampling frequency and stair wear (see Figure 6).

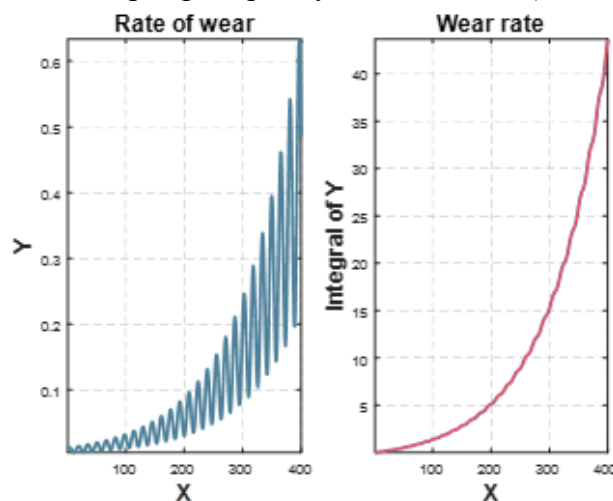


Figure 6 The Influence of Different Frequency Trampling on Stair Wear

As shown in Figure 6, this model can predict stair wear based on the trampling frequency: if high-frequency trampling occurs, the wear rate fluctuates and increases. At the initial stage, the material has high wear resistance, and the corrosion rate increases gently. After 200 years, the wear resistance decreased significantly and the corrosion rate increased exponentially. The fluctuation of wear speed reflects the dynamic response characteristics of materials.

In the stair wear simulation model based on finite element analysis, different elements will greatly interfere with the accuracy of the model. Therefore, this paper evaluates the stability of finite element calculation by replacing different elements and giving the number of elements when stress converges. There is a negative correlation between the number of elements and the stability of the calculation. LINK32 element used in this model has the least element demand and the best calculation efficiency

in stability calculation. Its strong adaptability to the isotropic rectangular stress field shows that the convergence stability of the finite element calculation model meets the accuracy requirements.

4. Conclusions

In this study, a finite element fatigue and wear model for historical building stairs is constructed through parametric simulation of materials and multi-scale mechanical analysis. The model realizes the dynamic coupling simulation of fatigue damage and wear process under periodic load and can predict the service life of stairs, repair history, building materials properties, trampling frequency, and other information according to the recorded wear information of stairs. The model adopts the cooperative calculation framework of the S-N fatigue curve and Archard's wear law and combines fixed boundary conditions and periodic load constraints to accurately characterize the cross-scale response law of material micro-defect evolution and macro-wear morphology. Through the joint verification of fitting test and difference test, the model effectively identifies the statistical difference between natural wear and artificial repair marks and reveals the anti-wear advantages of gneiss, serpentine, and other stones due to their high shear modulus and low fatigue sensitivity, and their wear rate is 1-2 orders of magnitude lower than that of Begonia and hard maple.

The innovation of this study lies in the integration of finite element analysis and material fatigue theory, and the interdisciplinary quantitative analysis paradigm of material mechanics and archaeology is established. By introducing confidence interval analysis and Monte Carlo random sampling, the robustness of the model to the uncertainty of historical data is significantly improved, which provides a scientific basis for the decision-making of cultural relics restoration based on the stress-life curve. Compared with the traditional qualitative description method of archaeology, this model realizes the dynamic reconstruction of the use mode of architectural heritage by reverse inversion of the mapping relationship between wear characteristics and load history and provides a new path for micro-history research and preventive protection technology.

Follow-up studies on the compatibility of the wear mechanism of the quasi-extended model, including mechanical fatigue wear, oxidation wear, and abrasive wear into a unified analysis framework, and integrated into a light reflection depression search algorithm^[11]. Through the three-dimensional shape reconstruction of high-resolution images and the quantification of bilateral wear difference rate, the historical pedestrian flow distribution characteristics can be accurately inverted. At the same time, the collaborative application of multi-physical field coupling simulation and machine learning optimization algorithm is explored to build a life cycle assessment system covering material degradation, environmental erosion, and structural response, and further improve the systematicness and scientificity of historical building protection technology.

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