ISSN:2790-1688 Volume-14-(2025)

Nonlinear Trajectory Tracking Control of Unmanned Vehicle based on Dynamic Linearization

Xuanzuo He *

School of Mechanical Engineering, Beijing Institute of Technology, Beijing, China.

1120232875@bit.edu.cn

Abstract. Aimed at the trajectory tracking control problem of nonlinear unmanned vehicle systems with unknown models, a model-free adaptive control algorithm based on dynamic linearization technology is proposed. Firstly, the dynamic data mapping of unmanned vehicles is established by the dynamic linearization method, and the dynamic of unmanned vehicles is estimated in real-time based on a pseudo-partial derivative estimation algorithm, which effectively reduces the modelling error of fixed parameters. On this basis, a model-free adaptive control scheme based on pseudo-partial derivatives is proposed, and the convergence of the algorithm is analyzed based on the compression mapping criterion. Finally, the effectiveness and robustness of the algorithm are verified by simulation experiments in normal and communication interruption scenarios.

Keywords: Dynamic linearization technology; Nonlinear unmanned vehicle; Data loss; Trajectory tracking.

1. Introduction

In recent years, nonlinear control systems have been widely used in the field of unmanned vehicles, including multi-vehicle cooperation and formation, robust control in complex environments, sensor fusion and state estimation. This paper focuses on the control method of unmanned vehicles. Because of its advantages in efficiency, safety, cost and accuracy, unmanned vehicle control technology is widely used in logistics and transportation, forestry inspection, self-driving taxis and other aspects. The key significance of unmanned vehicle control lies in improving system efficiency, enhancing safety performance and completing precise control.

Sun et al. [1] proposed a linear data model based on a communicative intelligent system and designed a model-free adaptive iterative learning consensus algorithm on this basis. Hou et al. [2] proposed a model-free adaptive control for discrete-time single-input-output nonlinear systems. Chen et al[3] analyzed the situation of actuator failure and bidirectional data packet loss and involved a new adaptive control strategy. Wang et al [4] investigated the problem of noise-to-state real finite-time output tracking control for stochastic nonlinear systems with mismatched disturbances. Deng et al. [5] studied the trajectory-tracking control problem of a class of disturbed general linear systems under the condition of event-triggered communication. Aiming at the containment control problem of discrete-time linear systems, Cao et al. [6] proposed a new model-free event-triggering method considering the quadratic index. Jiang et al. [7] proposed a trajectory-tracking controller for a class of nonlinear systems under directed graphs. Considering three different types of network attacks, Pan et al. [8] developed a unified data-driven control method without modelling. Focusing on the trajectory tracking problem of nonlinear systems with communication delay, Huang et al[9] proposed a distributed predictive control scheme. Li et al. [10] proposed a distributed output feedback model predictive control scheme for nonlinear system security. Li and Hou [11] studied the problem of event-triggered model-free adaptive predictive control for a class of networked nonlinear control systems attacked by deception. Chi et al. [12] proposed a new data-driven finite iterative learning control method for nonlinear repetitive systems that are stable in finite running time.

Sun et al. [1] did not involve the network structure that is not strongly connected or dynamically changed. Chen et al. [3] proposed that it was possible to involve an iterative learning controller when the input and output of the system are limited. Wang et al. [4] have not extended the results to systems with incomplete information. In the future, Cao et al. [6] will study how to balance optimality and trigger frequency in optimal event trigger control. Pan et al. [8] have not solved the consistency of model-free adaptive control for multi-input multi-output multi-nonlinear systems. According to Li et al. [10], if the research system cannot obtain accurate information about the leadership system, its proposed cost function will be invalid. Chi et al. [12] proposed to optimize DDFILC so that its selection has nothing to do with the initial state, which is a future research direction.

Inspired by the above problems, this paper studies the model-free adaptive control problem for unmanned vehicle trajectory tracking control, which is of great significance when the system model is unknown or uncertain. Firstly, the mathematical model of unmanned vehicle systems is established by dynamic linearization. On this basis, a pseudo-partial derivative estimation algorithm is proposed.

The main contributions of this paper are summarized as follows:

(1) The data-driven model-free control scheme is adopted, and the internal laws of the system are established only by input and output data, which overcomes the dependence of previous algorithms on the dynamic information of unmanned vehicles.

(2) The adaptive dynamic estimation of time-varying parameters is tested, which improves the control accuracy of the algorithm compared with the fixed parameter values in the traditional algorithm framework.

The following structure of this paper is as follows: the second section introduces the model-free adaptive control algorithm. In the third section, the stability analysis and proof of the algorithm are given. In the fourth section, the effectiveness of the algorithm is proved by simulation experiments, and the self-adjustment method of the algorithm is added in the case of communication interruption; The fifth section is a summary of this paper.

2. Problem Formulation

Considering the following single-input single-output unmanned vehicle system:

$$p\left(t+1\right)=f\left(p\left(t\right),i\left(t\right)\right), \tag{1}$$

Where, $i\left(t\right)\in R$ and p(t)∈R represent the energy input and driving distance of the unmanned vehicle at time t, respectively; f (⋯): R2→R is an unknown nonlinear function. The following hypotheses are put forward for the unmanned vehicle system.

Hypothesis 2.1: The partial derivative of f(⋯) with respect to the second variable is continuous except at finite moments.

Hypothesis 2.2: Except for finite time points, the unmanned vehicle (1) satisfies the generalized Lipschiz condition, that is, for any $t_1\neq t_2,t_1,t_2\geq 0$ and $i\left(t_1\right)\neq i\left(t_2\right)$, there is:

$$\left|p\left(t_1+1\right)-p(t_2+1)\right|\leq n\left|i(t_1)-i(t_2)\right|,$$

Where $p(t_j+1)=f(p(t_j),i(t_j)), j=1,2;n>0$ is a constant.

$\Delta p(t+1)=p(t+1)-p(t)$ is recorded as the change of driving distance at adjacent moments and $\Delta i(t+1)=i(t+1)-i(t)$ is the change of energy input at two adjacent moments. In Theorem 2.1, for the unmanned vehicle (1) satisfying hypotheses 2.1 and 2.2, there must be a time-varying parameter $\alpha(t)\in R$ called PPD at that time $\left|\Delta i(t)\neq 0\right|$, which makes the unmanned vehicle data mapping (1) be transformed into the following CFDL data model:

$$\Delta p(t+1)=\alpha(t)\Delta i(t), \tag{2}$$

And $\alpha(t)$ is bounded at any time t. According to Theorem 2.1, when the unmanned vehicle data map (1) satisfies hypotheses 2.1 and 2.2, and $\Delta i(t)\neq 0$ holds at any time t, its CFDL data model can be expressed as

$$p(t+1)=p(t)+\alpha(t)\Delta i(t), \tag{3}$$

Wherein, $\alpha(t)\in R$ is the PPD of the unmanned vehicle (1).

Considering the following criteria function:

$$J(i(t)) = \left|p^*(t+1) - p(t+1)\right|^2 + \beta\left|i(t) - i(t-1)\right|^2, \quad (4)$$

Where: $\beta > 0$ is a weight factor used to limit the change of control input energy; $p^*(t+1)$ is the expected driving distance for unmanned vehicles. Substituting the formula (3) into the formula (4), taking the derivative of $i(t)$ and making it equal to zero, we can get:

$$i(t) = i(t-1) + \frac{\gamma\alpha(t)}{\beta + |\alpha(t)|^2}\left[p^*(t+1) - p(t)\right], \quad (5)$$

Where: $\gamma \in (0,1]$ is the step factor, which makes the control algorithm more general. The following PDD criterion function is proposed:

$$J(\hat{\alpha}(t)) = \left|p(t) - p(t-1) - \hat{\alpha}(t)\Delta i(t-1)\right|^2 + \theta\left|\hat{\alpha}(t) - \hat{\alpha}(t-1)\right|^2, \quad (6)$$

Where $\theta > 0$ is the weight factor. Finding the extreme value of equation (6) about $\hat{\alpha}(t)$, the PDD estimation algorithm is as follows:

$$\hat{\alpha}(t) = \hat{\alpha}(t-1) + \frac{\mu\Delta i(t-1)}{\theta + \Delta i(t-1)^2}\left[\Delta p(t) - \hat{\alpha}(t-1)\Delta i(t-1)\right], \quad (7)$$

Where $\mu \in (0,1]$ is the step factor, which makes the algorithm more flexible and $\hat{\alpha}(t)$ is the estimated value of PDD $\alpha(t)$.

3. Stability Analysis

Hypothesis 3.1: For a given bounded expected driving distance $p^*(t+1)$, there is always a bounded $i^*(t)$, so that the driving distance of the unmanned vehicle (1) is equal to $p^*(t+1)$.

Hypothesis 3.2: For any time t and $\Delta i(t) \neq 0$, the symbol of the system PDD remains unchanged.

For the unmanned vehicle (1), under the conditions that hypotheses 2.1, 2.2, 3.1 and 3.2 are satisfied, when $p^*(t+1) = p^*$ is a constant, there is a positive integer $\beta_{\min} > 0$, so that there is $\beta > \beta_{\min}$:

(1) The tracking error of the driving distance of unmanned vehicles is monotonically convergent, and. $\lim_{t\to\infty}\left|p^* - p^*(t+1)\right| = 0$.

(2) The closed-loop system is BIBO stable, that is, the driving distance sequence $\{p(t)\}$ and energy input sequence $\{i(t)\}$ are bounded. The proof is as follows: $\tilde{\alpha}(t) = \hat{\alpha}(t) - \alpha(t)$ is defined as PDD estimation error. Subtracting $\alpha(t)$ at both sides of the parameter estimation algorithm (7) to obtain:

$$\begin{aligned}
\hat{\alpha}(t) - \alpha(t) &= \hat{\alpha}(t-1) - \alpha(t) + \alpha(t-1) - \alpha(t-1) + \frac{\mu\Delta i(t-1)}{\theta + \Delta i(t-1)^2}\left[\Delta p(t) - \hat{\alpha}(t-1)\Delta i(t-1)\right] \\
&= \tilde{\alpha}(t-1) - \alpha(t) + \alpha(t-1) + \frac{\mu\Delta i(t-1)}{\theta + \Delta i(t-1)^2}\left[\alpha(t-1)\Delta i(t-1) - \hat{\alpha}(t-1)\Delta i(t-1)\right] \\
&= \tilde{\alpha}(t-1) - \alpha(t) + \alpha(t-1) + \frac{\mu\Delta i(t-1)}{\theta + \Delta i(t-1)^2}\Delta i(t-1)\left[\alpha(t-1) - \hat{\alpha}(t-1)\right] \\
&= \tilde{\alpha}(t-1) - \alpha(t) + \alpha(t-1) + \frac{\mu\Delta i(t-1)^2}{\theta + \Delta i(t-1)^2}\left[-\tilde{\alpha}(t-1)\right]
\end{aligned}$$

To obtain:

$$\tilde{\alpha}(t)=\tilde{\alpha}(t-1)[1-\frac{\mu\Delta i(t-1)^2}{\theta+\Delta i(t-1)^2}]--\alpha(t)+\alpha(t-1),\tag{8}$$

By taking the absolute value of both sides of (8), we can get

$$\left|\tilde{\alpha}(t)\right|\le\left|\tilde{\alpha}(t-1)\right|\left|1-\frac{\mu\Delta i(t-1)^2}{\theta+\Delta i(t-1)^2}\right|+\left|\alpha(t-1)-\alpha(t)\right|,\tag{9}$$

Note that $\frac{\mu\Delta i(t-1)^2}{\theta+\Delta i(t-1)^2}$ monotonically increases with respect to $\Delta i(t-1)^2$, and its minimum value is $\frac{\mu\delta^2}{\theta+\delta^2}$. When $0<\mu\le 1$ and $\theta>0$, there is a constant k_1 that satisfies:

$$0\le\left|1-\frac{\mu\Delta i(t-1)^2}{\theta+\Delta i(t-1)^2}\right|\le\left|1-\frac{\mu\delta^2}{\theta+\delta^2}\right|=k_1<1,\tag{10}$$

According to Theorem 2.1, there is $\left|\alpha(t)\right|\le\overline{n}$ and we can get $\left|\alpha(t-1)-\alpha(t)\right|\le 2\overline{n}$.

It can be obtained from formula (9) and formula (10)

$$\begin{aligned}\left|\tilde{\alpha}(t)\right|&\le k_1\left|\tilde{\alpha}(t-1)\right|+2\overline{n}\le k_1^{\,2}\left|\tilde{\alpha}(t-2)\right|+2k_1\overline{n}+2\overline{n}\\&\le\cdots\le k_1^{\,t-1}\left|\tilde{\alpha}(1)\right|+2\overline{n}(1+k_1+k_1^{\,2}+\cdots+k_1^{\,t-2})\end{aligned}$$

Namely

$$\left|\tilde{\alpha}(t)\right|\le k_1^{\,t-1}\left|\tilde{\alpha}(1)\right|+2\overline{n}\frac{1-k_1^{\,t-1}}{1-k_1},\tag{11}$$

So $\tilde{\alpha}(t)$ is bounded, and because $\alpha(t)$ is bounded, it can be known that $\hat{\alpha}(t)$ is bounded.

Defining the tracking error of driving distance of unmanned vehicle.

$$m(t+1)=p^*-p(t+1),\tag{12}$$

Substituting the formula (3) into the formula (12) and taking the absolute values of both sides at the same time, we can obtain:

$$\begin{aligned}\left|m(t+1)\right|&=\left|p^*-p(t+1)\right|=\left|p^*(t+1)-p^*(t)+p^*(t)-p(t)-\alpha(t)\Delta i(t)\right|\\&=\left|\Delta p^*(t+1)+m(t)-\alpha(t)\frac{\gamma\hat{\alpha}(t)}{\beta+\left|\hat{\alpha}(t)\right|^2}m(t)\right|\\&\le\left|\Delta p^*(t+1)\right|+\left|m(t)\right|\left|1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+\left|\hat{\alpha}(t)\right|^2}\right|\end{aligned}$$

Namely

$$\left|m(t+1)\right|\le\left|m(t)\right|\left|1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+\left|\hat{\alpha}(t)\right|^2}\right|,\tag{13}$$

Making $\beta_{\min}=\frac{\overline{n}^2}{4}$, from the inequality $a^2+b^2\ge 2ab$, hypothesis 3.2, the guarantee condition of the reset algorithm and the $\hat{\alpha}(t)$-bounded proof, if selected $\beta>\beta_{\min}$, there must be a constant $0<W<1$, so that the following formula holds.

$$0<W\le\frac{\hat{\alpha}(t)\alpha(t)}{\beta+\left|\hat{\alpha}(t)\right|^2}\le\frac{\overline{n}\hat{\alpha}(t)}{\beta+\left|\hat{\alpha}(t)\right|^2}\le\frac{\overline{n}\hat{\alpha}(t)}{2\sqrt{\beta}\hat{\alpha}(t)}\le\frac{\overline{n}}{2\sqrt{\beta}}\le\frac{\overline{n}}{2\sqrt{\beta_{\min}}}=1,\tag{14}$$

There exists $k_2 < 1$ and make

$$\left|1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+|\hat{\alpha}(t)|^2}\right| = 1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+|\hat{\alpha}(t)|^2} \le 1-\gamma W = k_2 < 1, \tag{15}$$

Combining formula (13) and formula (15), we can get

$$|m(t+1)| \le k_2|m(t)| \le k_2{}^2|m(t-1)| \le \cdots \le k_2{}^t|m(1)|, \tag{16}$$

It means that conclusion (1) holds. When $p^*(t+1)$ is not a constant, the conclusion still holds, which is proved as follows:

Substituting the formula (3) into the formula (12) and taking the absolute values of both sides at the same time, we can obtain:

$$\begin{aligned}|m(t+1)| &= |p^* - p(t+1)| = |p^* - p(t) - \alpha(t)\Delta i(t)| = |\Delta p^*(t+1) + m(t) - \alpha(t)\Delta i(t)| \\ &= \left|\Delta p^*(t+1) + m(t) - \alpha(t)\frac{\gamma\hat{\alpha}(t)}{\beta+|\hat{\alpha}(t)|^2}m(t)\right| \\ &\le |\Delta p^*(t+1)| + |m(t)|\left|1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+|\hat{\alpha}(t)|^2}\right|\end{aligned}$$

As we know that $p^*(t+1)$ is bounded, we make $|\Delta p^*(t+1)| \le n$. According to formula (15), we can know $\left|1-\frac{\gamma\hat{\alpha}(t)\alpha(t)}{\beta+|\hat{\alpha}(t)|^2}\right| \le k_2 < 1$. Substituting it into equation (3.10), we can obtain:

$$\begin{aligned}|m(t+1)| &\le n + k_2|m(t)| \le n + k_2[n + k_2|m(t-1)|] \\ &\le k_2{}^2|m(t-1)| + n + k_2 n \le \cdots \le k_2{}^t|m(1)| + n(1 + k_2 + k_2{}^2 + \cdots k_2{}^{t-1}) \\ &\le k_2{}^t|m(1)| + n\frac{1-k_2{}^t}{1-k_2} = \frac{n}{1-k_2}\end{aligned}$$

4. Simulation

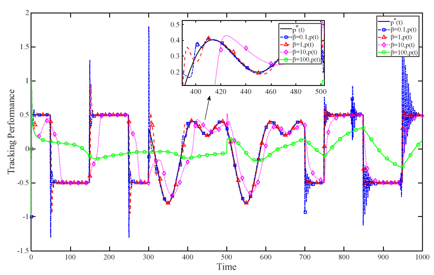

Figure 4-1: Tracking Performance

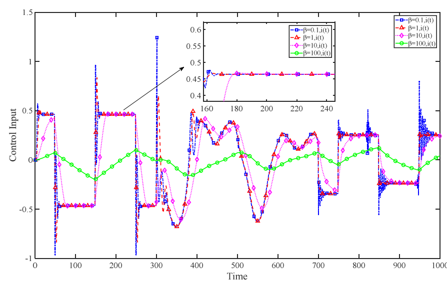

Figure 4-2: Control Input

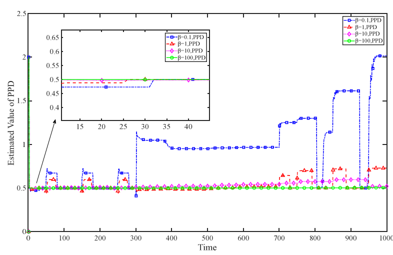

Figure 4-3: PDD Estimation

As can be seen from Figure 4-1, the smaller the parameter β, the better the tracking performance of the unmanned vehicle. As can be seen from Figure 4-2, the greater β, the smaller the control input, the less energy the unmanned vehicle consumes. As can be seen from Figure 4-3, with the increase of time, the larger the β, the more stable the PDD estimated value.

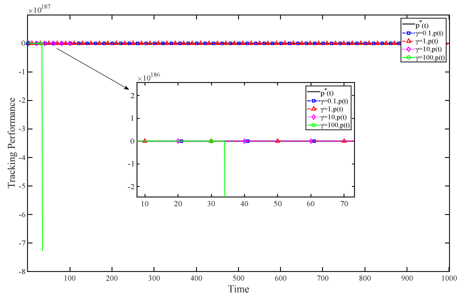

Figure 4-4: Tracking Performance

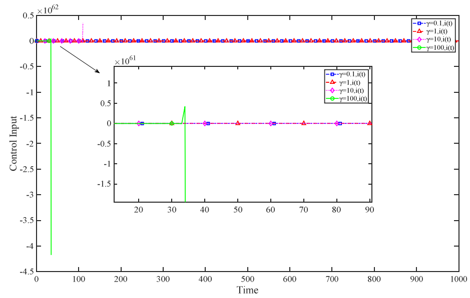

Figure 4-5: Control Input

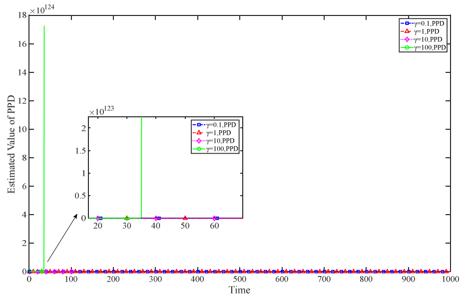

Figure 4-6: PPD Estimation

As shown in Figures 4-4, Figure 4-5 and Figure 4-6, the parameter γ has little influence on the tracking performance, energy input and PDD estimation of the unmanned vehicle.

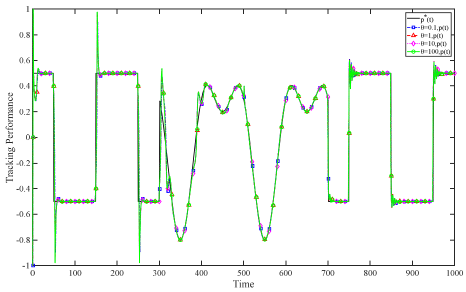

Figure 4-7: Tracking Performance

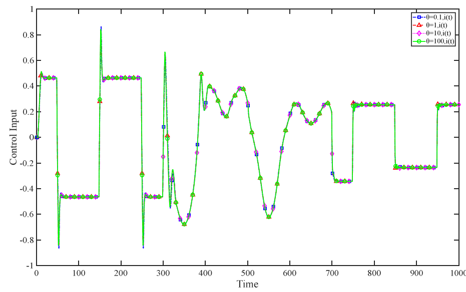

Figure 4-8: Control Input

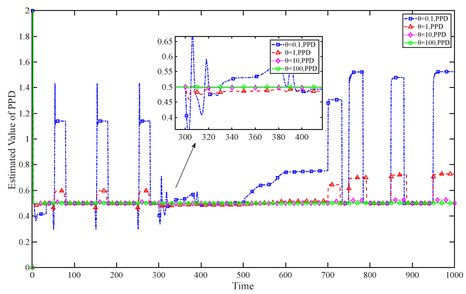

Figure 4-9: PDD Estimation

As shown in Figure 4-7 and Figure 4-8, the tracking performance and control input of the unmanned vehicle are more robust than the parameter θ. As shown in Figure 4-9, the larger the parameter θ, the more stable the PDD estimation of unmanned vehicles.

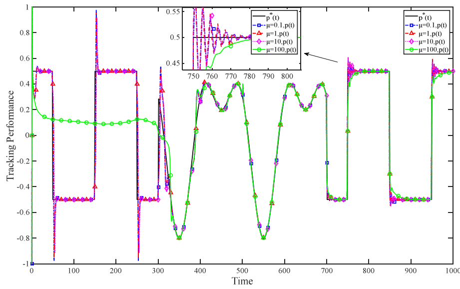

Figure 4-10: Tracking Performance

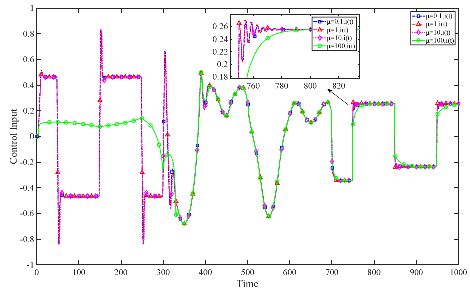

Figure 4-11: Control Input

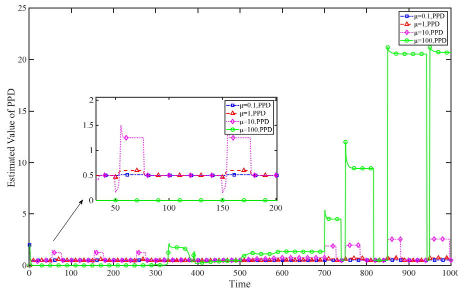

Figure 4-12: PPD Estimation

As can be seen from Figure 4-10 and Figure 4-11, with the increase of time, the influence of parameters μ on the tracking performance and energy input of unmanned vehicles becomes smaller and can be ignored. As shown in Figure 4-12, the smaller the parameter μ, the more stable the PDD estimation of unmanned vehicles.

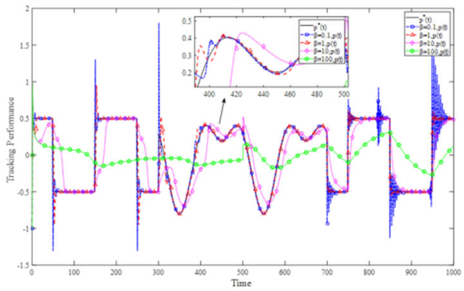

Figure 4-13: Normal Tracking Performance

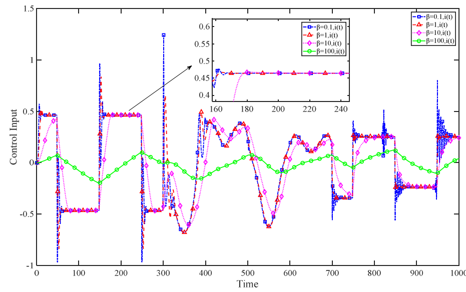

Figure 4-14: Normal Control Input

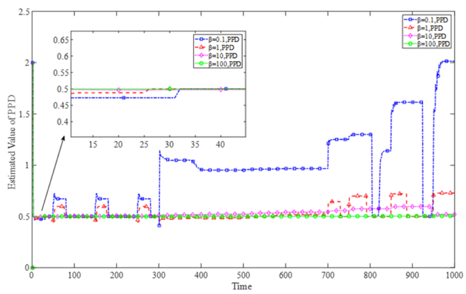

Figure 4-15: Normal PPD Estimation

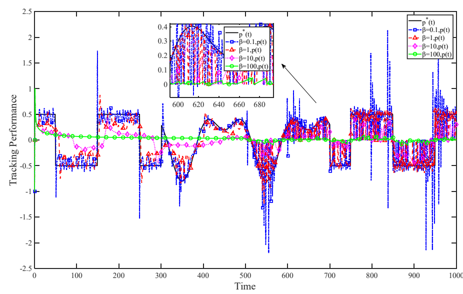

Figure 4-16: Random i(t)=0 Tracking Performance

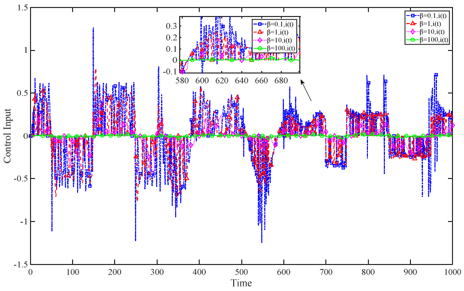

Figure 4-17: Random i(t)=0 Control Input

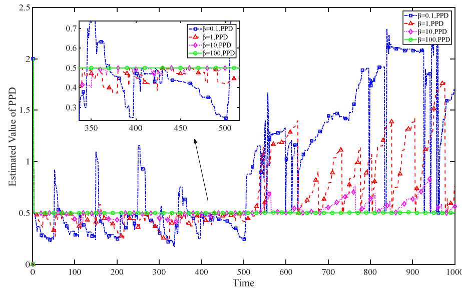

Figure 4-18: Random i(t)=0 PDD Estimation

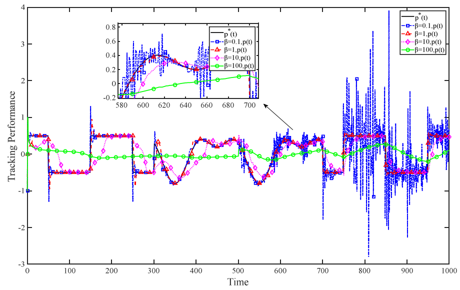

Figure 4-19: Tracking Performance with Compensation

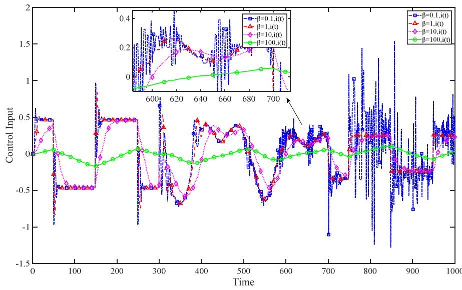

Figure 4-20: Control Input with Compensation

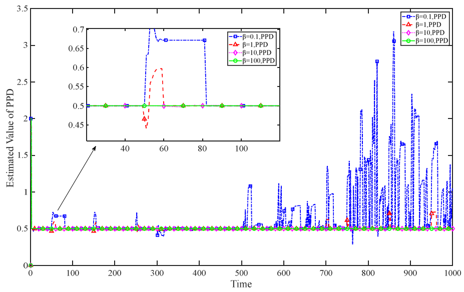

Figure 4-21: Input PPD Estimation with Compensation

Comparing Figure 4-13, Figure 4-16 and Figure 4-19, we can see that supplementing $i(t) = i(t-1)$ can reduce the influence of poor tracking performance caused by input loss to a certain extent. Comparing Figure 4-14, Figure 4-17 and Figure 4-20, it can be seen that supplementing $i(t) = i(t-1)$, in the case of input loss, the input energy of unmanned vehicles will not fluctuate greatly. By comparing Figure 4-15, Figure 4-18 and Figure 4-21, it can be seen that supplementing $i(t) = i(t-1)$ can stabilize the PDD estimation of unmanned vehicles to a certain extent.

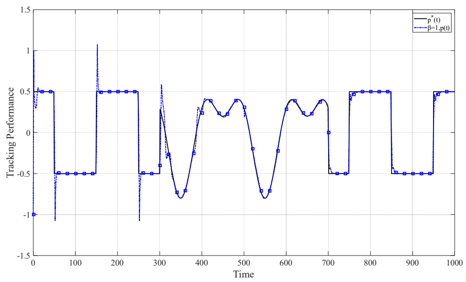

Figure 4-22: Tracking Performance under PID

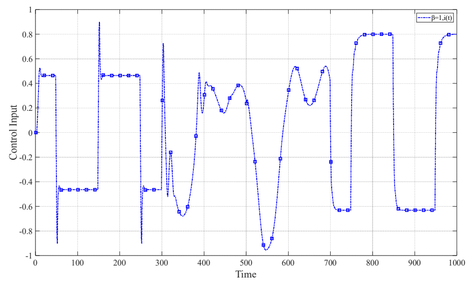

Figure 4-23: Control Input under PID

Comparing Figure 4-13 with Figure 4-22, Figure 4-14 and Figure 4-23, it is found that this set of algorithms can achieve better control effects through data driving without the model.

5. Conclusion

Aimed at the model-free control problem of nonlinear unmanned vehicle systems, this paper proposes a data-driven adaptive algorithm, which realizes efficient trajectory tracking through dynamic linearization and PPD estimation. The stability of the algorithm is proved theoretically, and its effectiveness is verified by simulation experiments under different parameters and communication interruption scenarios. Future work can be combined with deep learning methods to enhance environmental adaptability and explore resource optimization in hardware deployment.

References

[1] S. Sun, R. Chi, Y. Liu and N. Lin, "Model-free adaptive iterative learning from communicable agents for nonlinear networks consensus, " IEEE Transactions on Signal and Information Processing over Networks, vol. 9, pp. 458-467, 2023.

[2] Z. Hou and S. Jin, "A novel data-driven control approach for a class of discrete-time nonlinear systems," IEEE Transactions on Control Systems Technology, vol. 19, no. 6, pp. 1549-1558, Nov. 2011.

[3] J. Chen, C. Hua and X. Guan, "Iterative learning model-free control for networked systems with dual-direction data dropouts and actuator faults, " IEEE Transactions on Neural Networks and Learning Systems, vol. 32, no. 11, pp. 5232-5240, Nov. 2021.

[4] F. Wang, J. Gong, Z. Liu and Z. Chen, "Finite-time output tracking control for random multi-agent systems with mismatched disturbances, " IEEE Transactions on Automation Science and Engineering, vol. 21, no. 4, pp. 7516-7526, Oct. 2024.

[5] J. Deng, F. Wang, Z. Liu and Z. Chen, "Fully distributed consensus control for a class of disturbed linear multi-agent systems over event-triggered communication, " IEEE Transactions on Signal and Information Processing over Networks, vol. 10, pp. 205-215, 2024.

[6] A. Cao, F. Wang, Z. Liu and Z. Chen, "Model-free event-triggered optimal containment control for multiagent systems via adaptive dynamic programming, " IEEE Transactions on Control of Network Systems, vol. 11, no. 3, pp. 1452-1464, Sept. 2024.

[7] Y. Jiang, F. Wang, Z. Liu and Z. Chen, "Composite learning adaptive tracking control for full-state constrained multiagent systems without using the feasibility condition, " IEEE Transactions on Neural Networks and Learning Systems, vol. 35, no. 2, pp. 2460-2472, Feb. 2024.

[8] Z. Pan, R. Chi and Z. Hou, "Compensation-based distributed model-free adaptive control for cyber-attacks, " IEEE Transactions on Signal and Information Processing over Networks, vol. 9, pp. 84-94, 2023.

[9] Y. Huang, G. -P. Liu, Y. Yu and W. Hu, "Data-driven distributed predictive tracking control for heterogeneous nonlinear multiagent systems with communication delays, " IEEE Transactions on Automatic Control, vol. 69, no. 7, pp. 4786-4792, July 2024.

[10] Z. Li, Y. Shi, S. Xu, H. Xu and L. Dong, "Distributed model predictive consensus of mass against false data injection attacks and denial-of-service attacks," IEEE Transactions on Automatic Control, vol. 69, no. 8, pp. 5538-5545, Aug. 2024.

[11] F. Li and Z. Hou, "Event-triggered model-free adaptive predictive control for networked control systems under deception attacks, " IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 54, no. 2, pp. 1325-1334, Feb. 2024.

[12] R. Chi, Z. Liu, N. Lin, Z. Hou and B. Huang, "Data-driven finite-iteration learning control," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 54, no. 9, pp. 5296-5306, Sept. 2024.