

Application of mathematical analysis in intelligent driving safety warning system design

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Abstract. This paper explores the application of mathematical analysis in the design of intelligent driving safety warning systems, addressing requirements such as multi-dimensional environmental information perception fusion, vehicle motion trajectory prediction, collision risk assessment, and warning threshold setting. On this basis, a scientifically sound and reasonable intelligent driving safety warning system is constructed, fully demonstrating the importance and application of mathematical analysis.

Keywords: mathematical analysis; intelligent driving; safety warning system; design framework.

The intelligent driving safety warning system is a core technical module that ensures the safe operation of autonomous vehicles. The dynamic nature of complex traffic scenarios demands that the system possess millisecond-level risk identification and decision-making capabilities, for which mathematical analysis tools provide theoretical support. Probability theory and stochastic processes construct a framework for quantifying risks in uncertain environments, while differential equations describe the evolution laws of vehicle dynamics. Optimization theory supports the real-time generation of warning strategies[1]. Exploring the application paths of mathematical analysis in warning systems not only deepens and expands traditional control theory but also serves as a necessary foundation for achieving advanced autonomous driving. This paper constructs a system design framework driven by mathematical analysis to explore the integration path between theoretical tools and practical engineering needs.

1. Core requirements and challenges of intelligent driving safety warning system

1.1 Core requirements of the system

The intelligent driving safety warning system needs to achieve precise perception and integration of multi-dimensional environmental information. The spatiotemporal alignment requirement between LiDAR point cloud data and camera image data has led to the need for constructing a 3D spatial coordinate transformation model. Vehicle motion trajectory prediction requires establishing a differential equation model based on historical trajectory data to describe the continuous variation characteristics of parameters such as acceleration and steering angle [2]. Collision risk assessment must comprehensively consider the coupled effects of multiple variables, including relative speed, road adhesion coefficient, and braking response delay, which requires the system to have nonlinear optimization capabilities under multi-object constraints. The setting of warning thresholds must balance the contradiction between false alarm rate and missed detection rate, and the parameter calibration of probability density functions directly affects system reliability.

1.2 Technical challenges

Data noise interference in complex scenarios imposes robustness requirements on feature extraction algorithms. Sensor signal attenuation caused by rainy or foggy weather necessitates the design of adaptive filtering algorithms, and the covariance matrix update strategy of the Kalman

filter must be dynamically linked to environmental visibility. The fusion of heterogeneous sensor data faces timestamp asynchrony issues, and the choice of interpolation algorithms affects spatial registration accuracy. The real-time requirements of risk prediction models conflict with computational complexity, while the sampling efficiency of Monte Carlo simulations constrains long-term prediction capabilities. Decision optimization confronts dimensionality disaster in high-dimensional state spaces, and the computational time of dynamic programming algorithms grows exponentially with the number of variables, necessitating innovative applications of dimensionality reduction methods and approximation algorithms.

2. The key role of mathematical analysis tools in early warning system

2.1 Data preprocessing and feature extraction

The Fourier transform plays a central role in sensor signal denoising, with frequency domain filtering effectively separating raindrop noise from effective target reflection signals. The spatial clustering of point cloud data relies on density peak detection algorithms, and Gaussian mixture models excel in modeling obstacle shapes. Edge feature extraction from image data is based on Sobel operator gradient calculations, while mathematical optimization of convolution kernel parameters enhances feature discrimination. Time series alignment uses dynamic time warping algorithms to achieve multi-source data synchronization by minimizing path cost functions. Principal component analysis reduces data dimensions while retaining major variance information, and eigenvalue decomposition of the covariance matrix determines the projection direction.

2.2 Risk prediction and modeling

Vehicle trajectory prediction is modeled using stochastic differential equations, with Ito integral handling the uncertainties introduced by Brownian motion. The collision time calculation requires solving the motion equations of both vehicles simultaneously, and Newton's method accelerates the numerical solution convergence of nonlinear systems [3]. Risk probability assessment is based on Bayesian networks, where conditional probability tables reflect causal relationships between variables. The Markov decision process characterizes state transition patterns, and value iteration algorithms solve for optimal early warning strategies. Monte Carlo tree search balances exploration and exploitation, while the confidence upper limit formula guides path sampling directions.

2.3 Decision optimization and threshold design

Multi-objective optimization problems seek non-dominated solution sets through Pareto frontier analysis, with weight coefficients calibrated to reflect the trade-off between safety and comfort. Support vector machines construct classification hyperplanes, while kernel function selection influences the precision of high-risk area segmentation. Fuzzy logic handles uncertain inputs, and optimizing the shape parameters of membership functions enhances the adaptability of rule libraries. Dynamic threshold design employs sliding window statistics, and exponential weighted moving average models track environmental trends. Risk level classification is based on cluster analysis, and contour coefficients evaluate the tightness and separation of classification results.

3. Design framework of security early warning system driven by mathematical analysis

3.1 Hierarchical architecture design

3.1.1 Perception layer

The mathematical modeling process at the perception layer needs to address the spatiotemporal alignment of multi-source heterogeneous data. The point cloud data collected by LiDAR is spatially mapped through rigid transformation matrices, and the optimization calculation of rotation and translation parameters uses singular value decomposition. Quaternions represent rotations in three-dimensional space transformations, avoiding the universal joint lock phenomenon associated with Euler angles and maintaining numerical stability in rotation operations. Camera image distortion correction relies on polynomial fitting models, and the joint estimation of radial and tangential distortion coefficients is optimized using the least squares method to correct geometric distortions in images. Multi-target tracking algorithms integrate a joint probability data association framework, where the Hungarian algorithm solves the optimal matching problem of cost matrices to ensure the continuity of motion target IDs.

3.1.2 Decision makers

The mathematical analysis at the decision-making level constructs a closed-loop system for risk quantification and strategy generation. The potential field method in path planning constructs the superposition of obstacle repulsive fields and target attractive fields to form a potential energy. The gradient descent algorithm searches for collision-free paths along the negative gradient direction of the potential energy, avoiding local minima traps. Model predictive control employs a rolling time-domain optimization strategy, where the current state measurement initializes the prediction model. The optimal control sequence within a finite time domain is computed using a quadratic programming solver, and input-output constraints are embedded into the optimization objective through the Lagrangian multiplier method. The game theory model models human-vehicle interaction as an incomplete information dynamic game, with payoff functions quantifying the trade-off between maintaining safe distances and traffic efficiency. The backward induction method solves for refined Bayesian equilibrium strategies. Robust optimization methods handle parameter uncertainties by defining the fluctuation boundaries of state variables with box uncertainty sets, and the worst-case objective function is transformed into a solvable form through duality theory.

3.2 Real-time guarantee technology

3.2.1 Parallel computing acceleration

The design of parallel computing architectures needs to break through the efficiency bottlenecks of traditional serial algorithms. The theory of computational graph partitioning decomposes algorithms into independent sub-tasks, while critical path analysis identifies the core computational chains that affect overall latency. Task schedulers allocate computing resources based on priority. Matrix block operation strategies break down large matrices into sub-matrices suitable for GPU processing, with CUDA kernel functions optimizing memory access patterns and shared memory caching frequently used data blocks. Sparse matrix storage employs compressed row storage format, where the combination of row offset pointers and column index arrays significantly reduces memory usage. The zero element skipping strategy eliminates unnecessary computations in matrix multiplication. Pipeline technology overlaps stages such as data preprocessing, feature extraction,

and decision-making calculations. Dual-buffer mechanisms establish ping-pong operations between device memory and host memory, eliminating data transfer wait times.

3.2.2 Edge computing deployment

Resource management of edge computing nodes requires balancing computational load and communication latency. The task allocation problem is modeled as a multidimensional knapsack problem, where the greedy algorithm can solve an approximate optimal solution in polynomial time. CPU and GPU resource load balancing is dynamically adjusted through task weight coefficients. The task offloading decision model considers fluctuations in wireless channel conditions, and the Lyapunov optimization framework transforms long-term energy consumption constraints into instantaneous optimization problems. The virtual queue mechanism ensures the feasibility of time-averaged constraints. Knowledge distillation technology compresses deep neural network models, with soft labels from the teacher network passing implicit knowledge to the student network, which mimics the teacher's behavior through a KL divergence loss function. Incremental update strategies only transmit the differences in model parameters, while differential encoding algorithms leverage the sparsity of parameter changes, and Huffman coding further reduces packet size. Cache management employs a least recently used replacement strategy, with hash tables establishing a quick mapping between data block identifiers and storage locations, and Bloom filters accelerating cache lookup processes.

3.3 Case verification and parameter optimization

3.3.1 Typical scenario testing

The intersection scenario is modeled using a mixed-integer programming model, where binary variables represent access rights allocation. The lane-changing game establishes a non-cooperative game model, and the subgame perfect equilibrium is solved using backward induction. For the following scenario, a following differential equation is adopted, and the Lyapunov exponent evaluates the stability of the convoy. Random perturbations are introduced under extreme weather conditions, and Monte Carlo simulation assesses system robustness. The generalization capability of the scenario is verified through topological isomorphic mapping, and manifold learning maintains the invariance of the feature space structure.

3.3.2 Parameter sensitivity analysis

Global sensitivity analysis employs the Sobol index method, quantifying parameter interaction effects through variance decomposition. Local sensitivity analysis relies on partial derivative calculations, with the Jacobian matrix revealing the directional impact of parameter changes. Parameter optimization uses the particle swarm algorithm, with inertia weights balancing global search and local exploration. Bayesian optimization constructs surrogate models to guide the order of parameter space exploration through function collection. Hyperparameter tuning applies grid search methods, with cross-validation preventing overfitting.

Sum Up

Mathematical analysis tools provide comprehensive theoretical support for the intelligent driving safety warning system, from data perception to decision execution. Probability models quantify environmental uncertainties, optimization algorithms ensure real-time decision-making efficiency, and differential equations characterize dynamic evolution patterns. A hierarchical architecture

design decouples complex functions into modular components, while parallel computing and edge deployment address real-time performance bottlenecks. Parameter sensitivity analysis guides the enhancement of system robustness, and typical scenario validation improves functional coverage dimensions. Future research should further explore the application of quantum computing in optimization problems and the potential of differential geometry in modeling complex scenarios. The deep integration of mathematical theory and engineering practice will continue to drive the innovative development of intelligent driving safety technology.

Reference Documentation

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