

# A study of fast implementation methods for radar super-resolution imaging

Jinhui Xing <sup>1, a</sup>, Fangxing Yang <sup>1, b</sup>, Wei Zhao <sup>1, c</sup>, Changcheng Hu <sup>1, d</sup>

<sup>1</sup> School of Electronic Engineering and Optoelectronic Technology, Nanjing University of Science and Technology, China

<sup>a</sup>xingjinhui@njust.edu.cn, <sup>b</sup>2675458329@qq.com, <sup>c</sup>3110499665@qq.com,

<sup>d</sup>huchangcheng@njust.edu.cn

**Abstract.** In this paper, the radar forward-looking imaging are firstly established spatial geometry model and echo model and then the signal processing method is adopted to achieve azimuth super-resolution, and the regularization based inverse convolution method is used to construct the imaging objective function equation. In solving the regularization equation, an adaptive thresholding shrinkage algorithm with good convergence and noise resistance is used. In addition, an adaptive iterative extrapolation algorithm has been studied, which can effectively improve the convergence speed of the adaptive threshold shrinkage method.

**Keywords:** Forward-looking scanning, regularization, adaptive thresholding shrinkage, adaptive iterative extrapolation.

## 1. Forward-looking radar super-resolution imaging models

### 1.1 Echo Modeling

#### 1.1.1 Spatial Geometric Model

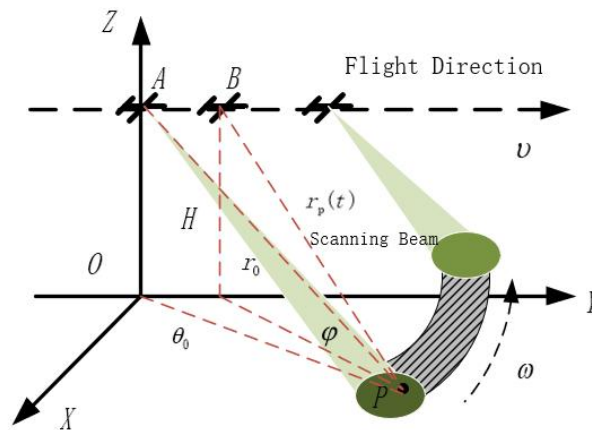


Figure 1 Spatial Geometry Mode

The spatial geometry model of the forward-looking scanning radar on a moving platform is shown in Figure 1. It is assumed that the platform has a height of  $H$ , a velocity of  $v$ , and a uniform linear motion along the  $Y$ -axis. The radar adopts a linear frequency modulation signal, and the pulse repetition frequency remains constant. The antenna performs forward-looking scanning with an angular velocity  $\omega$  in the counterclockwise direction[1,2,3].

#### 1.1.2 Signal Echo Modelling

In the case of a point target, the distance upward radar transmits a linear FM signal  $s(\tau)$  with a constant pulse repetition frequency with an initial slant distance of  $r_0$ , and a point-target scattering intensity of  $\sigma_0$ , and its target echo at the initial position A can be expressed as:

$$s_p(\tau) = \sigma_0 \text{rect} \left( \frac{\tau - 2r_0/c}{T_r} \right) \cos \left\{ 2\pi f_0 \left( \tau - \frac{2r_0}{c} \right) + \pi K_r \left( \tau - \frac{2r_0}{c} \right)^2 \right\} \quad (1-1)$$

where  $c$  is the speed of light.

## 1.2 Echo Preprocessing

On the basis of establishing the echo signal model, the echo is preprocessed by pulse compression and distance walk correction, and the convolution-like model under the motion platform is deduced to provide a research basis for the realization of image super-resolution.

### 1.2.1 Pulse Compression

The radar forward-looking scanning imaging system improves the distance resolution in pulse compression processing by linearly modulating the frequency signal and matched filtering. The specific method is to first transform the echo signal from the distance domain to the frequency domain, construct a matched filter function, and perform a product operation on the frequency domain to eliminate the quadratic phase and retain only the linear position phase. Subsequently, the signal is changed back to the distance domain by an inverse fast Fourier transform to obtain the pulse-compressed signal.

### 1.2.2 Distance walking correction

When a radar beam is fired, the platform motion causes a distance walk in the echo data. To eliminate this effect, the echo data without Doppler modulation can be recovered by applying a linear scale transform to the distance variable and correcting for the Doppler effect in the frequency domain by means of a Fourier transform and phase compensation. In addition, a constant matrix can be generated to estimate the effect of platform motion on the echo data and used for phase compensation.

### 1.2.3 Sparse-based adaptive threshold shrinkage regularization approach

Azimuth beam sharpening is a key step in achieving high-resolution forward-looking imaging[4,5]. It involves multiple preprocessing operations on the echo to transform the azimuth echo into a convolution-like process between the target scattering distribution and the antenna orientation map.

To facilitate digital signal processing, the signal and antenna radiation pattern are digitally discretized. The equation is as follows:

$$s_{N \times 1} = \tilde{H}_{N \times l} x_{M \times 1} \quad (1-2)$$

In Eq. (1-2)  $s$  is the orientation echo column vector with length  $N$ , indicating the number of orientation samples of the echo in the same distance cell;  $H$  is the antenna direction map column vector with length  $l$ , indicating the number of antenna direction map samples; and  $x$  is the target scattering column vector with length  $M$ , indicating the number of target points during convolution.

Since the number of unknowns  $M$  is larger than the number of equations  $N$ , in order to facilitate the inverse convolution process[6]. The first  $\tilde{H}_{N \times l}$  before  $L1$  columns are truncated, and the last  $L1$  columns are added to obtain the cyclic matrix  $H$ . The most direct way to get is to find the estimate of the target inverse matrix of the matrix  $H$ .

Regularization methods provide an effective approach to obtaining a stable solution. In the sparse based regularization method, the  $L1$  paradigm is added as a penalty to the objective function to constrain the solution and the estimator is given as follows

$$\hat{x} = \underset{x}{\operatorname{argmin}} [F(x) = \|s - Hx\|_2^2 + \lambda \|x\|_1] \quad (1-3)$$

This method suffers from a semi-convergence problem, meaning that under noise amplification, if the number of iterations is too high, the solution of the  $L1$ -norm constrained regularization method will rapidly deviate from the expected value.

To solve the unconstrained optimization problem in Equation (1-3), the adaptive thresholding shrinkage algorithm (ATSA) can be used. It has a simple structure and strong noise resistance. Firstly, the gradient of the objective function is calculated

$$\nabla F(x) = H^T(Hx - s) + \lambda \text{diag} \left[ \left( |x_i|^2 + \varepsilon \right)^{-\frac{1}{2}} \right] x \quad (1-4)$$

Where  $x_i$  is the  $i$ th element of  $x$  and  $\varepsilon$  is the diagonal matrix with the elements in the brackets as diagonals  $x$ . Then, the standard ATSA equation for obtaining the optimal solution is presented:

$$x_{k+1} = \mathfrak{R}_\delta \left[ x_k - \frac{1}{\beta} \nabla F(x) \right] \quad (1-5)$$

Where  $x_k$  is the iteration result after  $k$  iterations.  $\beta$  is a fixed step size which is restricted to be greater than  $\|H^T H\|/2$  to ensure convergence.  $\mathfrak{R}_\delta : \mathbb{R}^N \rightarrow \mathbb{R}^N$  is the contraction/threshold operator defined by

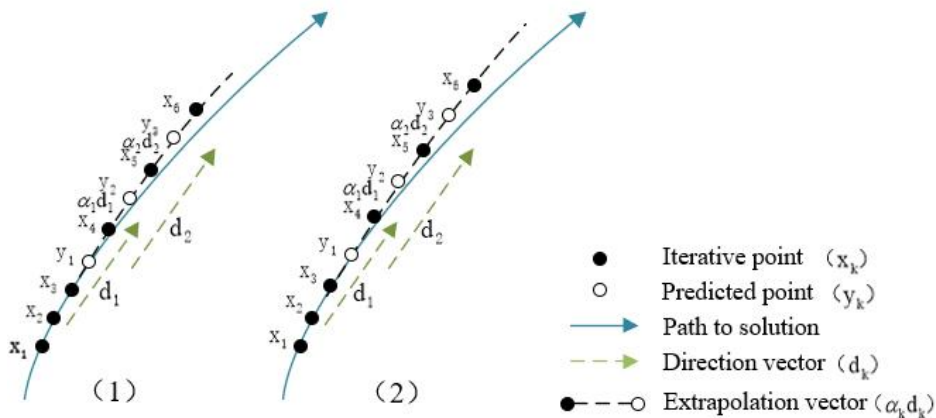
$$\mathfrak{R}_\delta(x) = [\mathfrak{R}_\delta(x_1), \dots, \mathfrak{R}_\delta(x_i), \dots, \mathfrak{R}_\delta(x_N)] \quad (1-6)$$

This operation can effectively reduce noise by eliminating small coefficients of the signal without much distortion of the real information. However, the speed of convergence greatly limits its use in real-time applications.

## 2. Adaptive Iterative Extrapolation Algorithm

### 2.1 Methodological derivation

To accelerate the ATSA, an adaptive iterative extrapolation algorithm (AIEA) is proposed. First, a prediction point is calculated using the direction vector between the current and previous iteration results, and operations are performed on it to enable the iteration point to advance rapidly along the convergence path. To further enhance acceleration performance, a longer direction vector is extrapolated to construct a farther prediction point[7]. The new method is shown in (2).



If  $x_k$  is the current iteration point and  $x_{k-1}$  and  $x_{k-2}$  are previous iteration points. Then the pointing vector can be determined by summing the two previous iteration vectors  $d_k$

$$d_k = (x_k - x_{k-1}) + (x_{k-1} - x_{k-2}) = x_k - x_{k-2} \quad (2-1)$$

Then, the prediction point is calculated by adaptively extrapolating a new direction vector based on the current iteration result.

$$y_{k+1} = x_k + \alpha_k d_k \quad (2-2)$$

Where  $\alpha_k$  is the acceleration step.

Finally, the adaptive iterative extrapolation acceleration method, as defined by the equation, is applied to iteratively process the new prediction points until the optimal solution is found:

$$x_{k+1} = \mathfrak{R}_\delta \left\{ y_{k+1} - \frac{1}{\beta} \left[ H^T(Hy_{k+1} - s) + \lambda \text{diag} \left\{ [|(y_{k+1})_i|^2 + \varepsilon]^{-\frac{1}{2}} \right\} y_{k+1} \right] \right\} \quad (2-3)$$

Since a longer directional vector is used for extrapolation, the iteration point can advance faster along the convergence path with the help of the prediction point, significantly reducing the number of iterations required for convergence.

### 3. Simulation Analysis

The simulation parameters are shown in Table 1, and the five target azimuths set are located at 4000m with angles of  $-5^\circ$  ,  $-2.5^\circ$  ,  $0^\circ$  ,  $2.5^\circ$  and  $5^\circ$  upwards.

Table 1. Target Parameters

Parameter	Value	Parameter	Value
Platform Speed (m/s)	200	Antenna Scanning Range ( $^\circ$ )	-10~10
Detection Range (m)	4000	Signal Bandwidth (MHz)	40
Scanning Speed ( $^\circ$ /s)	50 $^\circ$ /s	Signal Pulse Width ( $\mu$ s)	2
Pulse Repetition Frequency (Hz)	3000		

At SNR=30dB, the number of iterations is 500 simulation results are shown below:

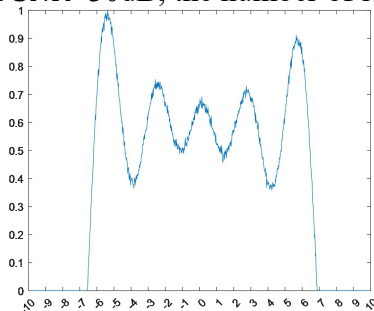


Figure 3.1 GD

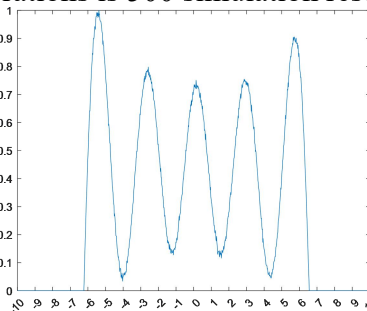


Figure3.2 ATSA

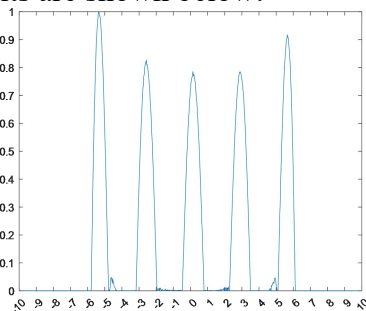


Figure3.3 AIEA

In order to quantitatively compare the performance of acceleration, the data error objective function  $J(\hat{\sigma})$  is used to compare the results with the true solution:

$$J(\hat{\sigma}) = \| S - H \hat{\sigma} \|_2^2 \tag{3-1}$$

As seen from the figure3.4, the Adaptive Iterative Extrapolation Acceleration Method converges the fastest, while the Gradient Descent Method is the slowest, with the Adaptive Threshold Shrinkage Method falling in between. After 1000 iterations, the normalized error for the Gradient Descent Method is 0.3865, for the Adaptive Threshold Shrinkage Method is 0.2492, and for the Adaptive Iterative Extrapolation Acceleration Method is 0.2670. The smaller the error, the closer the result is to the true value. Furthermore, Figure 3.5 and Table 2 show that when the normalized error approaches 0.3, the Adaptive Iterative Extrapolation Acceleration Method is significantly faster.

Table 2. Computation Time

Algorithm	Number of iterations	CPU time
GD	456	456
ATSA	789	213
AIEA	213	654

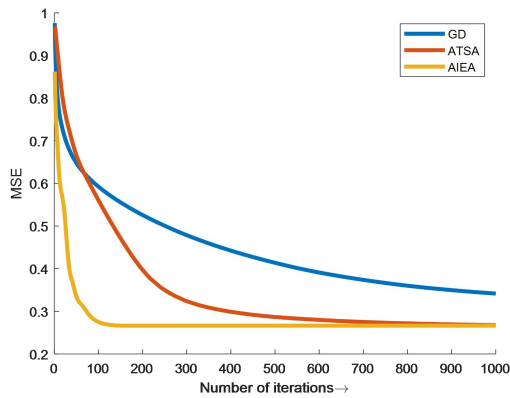


Figure 3.4

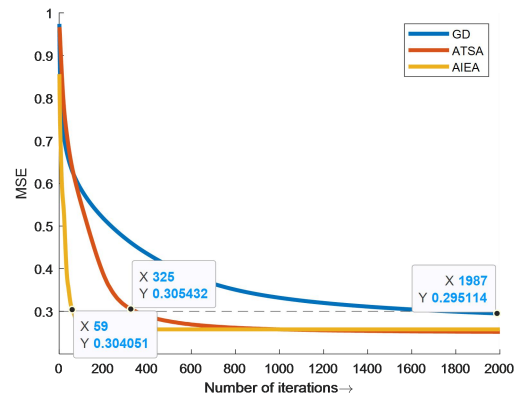


Figure 3.5

To demonstrate the noise resistance of the Adaptive Iterative Extrapolation Acceleration Method, the same sharpening process is applied to the echo at low signal-to-noise ratios. When the signal-to-noise ratio is 20dB and 10dB, the results are shown in the figure below.

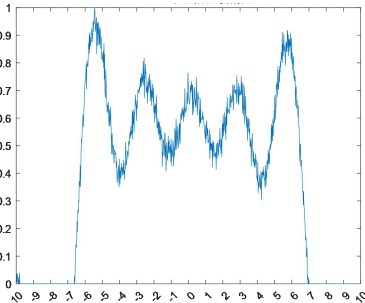


Figure3.6

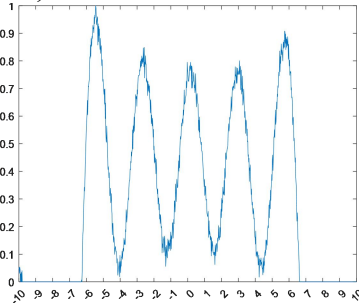


Figure3.7

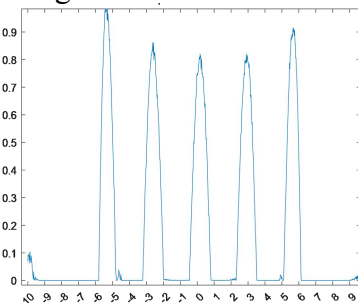


Figure3.8

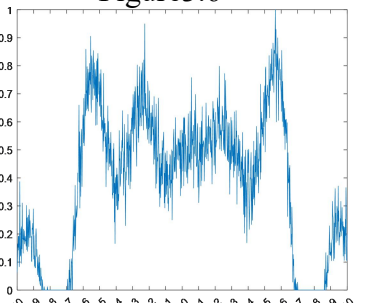


Figure3.9

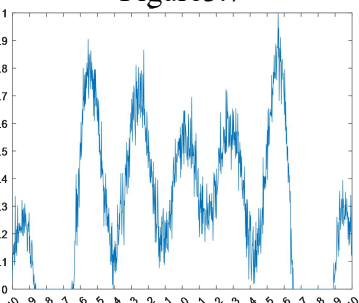


Figure3.10

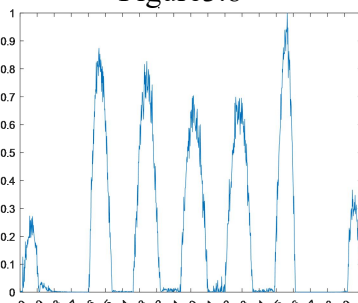


Figure3.11

It is evident that the results obtained under low SNR conditions exhibit significant noise. However, it can also be clearly observed that the gradient descent method results in pronounced edge noise, while the results obtained using the adaptive iterative extrapolation algorithm achieve better noise suppression, demonstrating its superior noise resistance.

#### 4. Summary

Through theoretical and simulation analysis, the adaptive iterative extrapolation algorithm demonstrates significant acceleration performance and noise resistance, highlighting its clear advantages. When the number of target signals is five and the angular separation is small, it achieves excellent resolution performance.

#### References

- [1] Mao DQ. Research on airborne radar scanning beam super-resolution imaging method[D]. University of Electronic Science and Technology,2022.
- [2] Qiu Dehou. A real-beam scanning radar echo modelling method[J]. Journal of Terahertz Science and Electronic Information,2019.

- [3] Zhang Yin. Theory and method of super-resolution imaging of forward-looking radar for motion platforms[D]. University of Electronic Science and Technology,2016.
- [4] TAN Ke. Research on scanning beam sharpening method of airborne forward-looking radar [D]. University of Electronic Science and Technology,2018.
- [5] Sha Liantong. Research and implementation of airborne radar super-resolution imaging method[D]. University of Electronic Science and Technology,2019.
- [6] Jingwen Ma. Research on radar corner super-resolution imaging method based on inverse convolution [D]. Xi'an Electronic Science and Technology University,2018.
- [7] Nie Xianbo. Research on fast implementation method of FPGA for super-resolution imaging[D]. University of Electronic Science and Technology,2020.