Application of the ERA Method in Vibration Testing of Frame-Structured Material Feeding Bridges Under Ambient Excitation

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Abstract. Ambient excitation modal analysis falls under operational modal analysis (OMA), requiring only knowledge of the output signals from the investigated structure during the analytical process. The Eigensystem Realization Algorithm (ERA) is a time-domain analysis method for conducting modal analysis of structures under ambient excitation. This paper elaborates on the fundamental principles and computational methodology of ERA. Through field data acquisition of acceleration response from a frame-structured material feeding bridge in a biomass power plant subjected to random equipment-induced vibrations, wavelet denoising is employed to extract the free vibration response signals of the structure, effectively separating structural dynamics from noise contamination, ultimately facilitating modal parameter extraction for the feeding bridge through the ERA. The Modal Amplitude Coherence (MAC) was introduced to eliminate spurious modes and discriminate authentic structural modal parameters. Subsequent cross-verification with finite element analysis (FEA) results demonstrates that the ERA algorithm achieves superior precision in identifying dynamic parameters of frame structures under ambient excitation.

Keywords: Ambient excitation, Frame-Structure, Eigensystem Realization Algorithm, modal identification, wavelet denoising techniques.

1. Introduction

The dynamic characteristics of structures—including natural frequencies, mode shapes, and damping ratios—have long been focal points of research and practical concern for civil engineering researchers and practitioners. A comprehensive understanding of these dynamic properties enables enhanced assessment of structural safety conditions and health status. As a prominent methodology for extracting structural dynamic characteristics, modal identification techniques have become indispensable in applications such as structural health monitoring, damage detection, and design optimization. Modal analysis encompasses two principal methodologies: [1] Operational Modal Analysis (OMA) and Experimental Modal Analysis (EMA). EMA, conducted under controlled laboratory conditions with precisely measured input excitation and output response data, is predominantly employed to validate technical accuracy through scaled structural models. This approach enables rigorous verification of modal identification techniques within deterministic experimental frameworks[2]. However, the majority of civil engineering structures are characterized by substantial dimensions, operational excitation difficulties, and prohibitive excitation costs. Consequently, operational modal analysis (OMA) remains the predominant methodology for modal characterization of such full-scale structures under ambient vibration conditions，Within

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this context, the predominant approaches involve operational modal analysis under ambient excitation and white noise sweep testing to extract structural dynamic parameters. Notable implementations include: Xu Liang's[3] identification of the first 10 natural frequencies below 50 Hz for the Humen Suspension Bridge using the Stochastic Subspace Identification (SSI) method, and Han Jianping's[4] successful extraction of the first four damping ratios for a rigid-frame composite bridge through the Natural Excitation Technique (NExT). In essence, modal parameter identification of structures under ambient excitation constitutes a computational technique that derives structural dynamic characteristics by isolating and analyzing free response components within operational vibration data. The Eigensystem Realization Algorithm (ERA) was originally proposed by Jer-Nan Juang[5] at the Langley Research Center of the (NASA). Integrating principles from control theory and observer design methodologies, this algorithm was initially applied to flight dynamics identification for aerospace vehicles, particularly in reconstructing orbital trajectories from limited sensor data. In 1995, Peterson pioneered the application of the Eigensystem Realization Algorithm (ERA) to civil engineering structures through a landmark study, demonstrating ERA's effectiveness and accuracy in extracting critical dynamic parameters, including natural frequencies and mode shapes.

2. ERA theoretical basis

2.1 Eigensystem Realization Algorithm（ERA）

For an n-dimensional linear time-invariant (LTI) system, its discrete-time state-space equations (taking a single-input single-output (SISO) system as an example) are expressed as:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \tag{1.1}$$

Let $x(k) \in R^n$ denote the n-dimensional state vector，$y(k) \in R^p$ the p-dimensional output vector, and $u(k) \in R^m$ the m-dimensional input vector, where k=1,2,3…represents the discrete-time sample index. The system matrix A, characterizing the dynamic properties of the system, encapsulates mass, stiffness, and damping information for elastic structures. The control matrix B, observation matrix C, and feedthrough matrix D complete the state-space representation. For systems undergoing free vibration or subjected to impulse excitation, equation (1.1) reduces to:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{1.2}$$

In discrete-time systems, the sequence of pulse response parameters governing the input-output relationship is formally defined as the Markov parameters. These parameters constitute the fundamental building blocks for system identification methodologies:

$$y(k) = CA^{k-1}B \tag{1.3}$$

For Multi-Input Multi-Output (MIMO) systems, the Markov parameters generalize to a sequence of matrix-valued impulse response coefficients. Specifically:

$$Y_k = CA^{k-1}B \tag{1.4}$$

Now，$Y_k = [y(k)_1, y(k)_2, y(k)_3 \cdots y(k)_q]$，q denote the number of output channels, where each component $y(k)_i$ of the output vector corresponds to a physical sensor measurement. The system realization is characterized by the triplet (A,B,C), where:

$A \in R^{n\times n}$ is the state matrix, $B \in R^{n\times m}$ is the control/input matrix, $C \in R^{p\times n}$ is the observation/output matrix.

For a controllable and observable system, the Eigensystem Realization Algorithm (ERA) enables robust identification of modal parameters by leveraging multiple response data samples. The methodology constructs a Hankel matrix from experimentally measured response data as follows:

$$H_{p\alpha\times ms}(k-1)=\begin{bmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+s-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+s} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+s-2} \end{bmatrix}_{p\alpha\times ms} \quad (1.5)$$

Performing Singular Value Decomposition (SVD) on the Hankel matrix H(0) yields::

$$H(0)=P_{p\alpha\times ms}\Sigma_{ms\times ms}U^T_{ms\times ms} \quad (1.6)$$

Where, $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_l \cdots 0 \cdots)$, and $\sigma_i > \sigma_{i+1}$, $i = 1,2,3\cdots$

The singular values of H(0), denoted as σ_i, are truncated by removing those close to zero, thereby determining the system order n:

$$\Sigma_n = diag(\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_n) \quad (1.7)$$

Truncate the first n columns of matrix P: $P_n = P[:,1:n]$, and the first n columns of matrix U: $U_n = U[:,1:n]$, assume $E_m = [I_{m\times m}\ \ 0_{m\times(s-1)m}]^T$, $E_p = [I_{p\times p}\ \ 0_{p\times(r-1)p}]^T$

The minimal realization of the system, represented by the matrices [A B C], is given as [5]:

$$\begin{cases} A=\Sigma_n^{-\frac{1}{2}}P_n^T H(1) U_n \Sigma_n^{-\frac{1}{2}} \\ B=\Sigma_n^{-\frac{1}{2}}U_n^T E_m \\ C=E_p^T P_n \Sigma_n^{-\frac{1}{2}} \end{cases} \quad (1.8)$$

The eigenvalue decomposition of the system matrix A as fallows:

$$A=\Phi Z \Phi^{-1} \quad (1.19)$$

Where $Z = \mathrm{diag}(z_1, z_2, z_3 \cdots z_n)$, and $\Phi = [\varphi_1, \varphi_2, \varphi_3 \cdots \varphi_n]$, for a discrete-time system, the eigenvalues and eigenvectors of A are generally complex-valued quantities. Based on the mapping between discrete-time systems and their continuous-time counterparts in the temporal domain, the modal parameters in the complex plane are defined as:

$$S_i=\frac{[lnz_i \pm 2\pi i]}{\Delta t}=\alpha_i \pm \beta_i j \quad (1.10)$$

The natural frequency of the system $f_i = \frac{\beta_i}{2\pi}$, and damping ratio $\varsigma_i = \frac{-\alpha_i}{\sqrt{\beta_i^2+\alpha_i^2}}$

2.2 MAC_i calculation

The modal discrimination index MAC_i is introduced to discriminate authentic structural modes from spurious noise-induced modes, enabling robust modal validation and noise rejection. [5]:

The modal amplitudes $\overline{Q_i}$ identified through singular value decomposition (SVD) of the Hankel matrix constructed from experimentally measured response data are quantified as:

$$\left[\overline{q}_1, \overline{q}_2, \overline{q}_3 \cdots \overline{q}_n\right] = \left[\Phi^{-1}\Sigma_n^{\frac{1}{2}}{U_n}^T\right]^* \tag{2.1}$$

In the complex plane, the theoretical modal amplitudes $\widehat{Q}_i$ for ideal vibration modes are defined as:

$$\hat{q}_i = \left[b_i^*, z_i b_i^*, z_i^2 b_i^* \cdots z_i^{s-1} b_i^*\right]^* \tag{2.2}$$

Where, $[b_1, b_2, b_3 \cdots b_n] = [\Phi^{-1}B]^*$, The superscript * denotes the conjugate transpose, the Modal Assurance Criterion (MAC) is defined as:

$$MAC_i = \frac{|\overline{q}_i^* \hat{q}_i|}{\sqrt{|\overline{q}_i^* \overline{q}_i|} \times \sqrt{|\hat{q}_i^* \hat{q}_i|}} \tag{2.3}$$

A Modal Amplitude Coherence MAC_i value approaching unity indicates that the identified mode closely approximates the ideal theoretical mode.

3. Project introduction and response testing

3.1 Structure overview

The vibration testing subject is a biomass power plant located in Xinglong Town, Bayan County, Heilongjiang Province. The facility complex comprises multiple structures including: Boiler house, Steam turbine hall, Office-residential complex, Material conveying trestle bridge, Cooling towers, Chimney stack. Of particular note, vibrations generated by the fuel feeding grate equipment within the boiler house induce perceptible vibrations across the aforementioned structures.

The tested material conveying trestle bridge features a reinforced concrete frame structure with the following geometric parameters: slope $\frac{h}{l} = \frac{21.4-4.69}{60.9} = 0.27$, length: 60.9 meters, width: 9.6 meters, Height: 21.4 meters. The trestle bridge is illustrated in Figure 1, while Table 1 provides detailed information on its primary structural components.

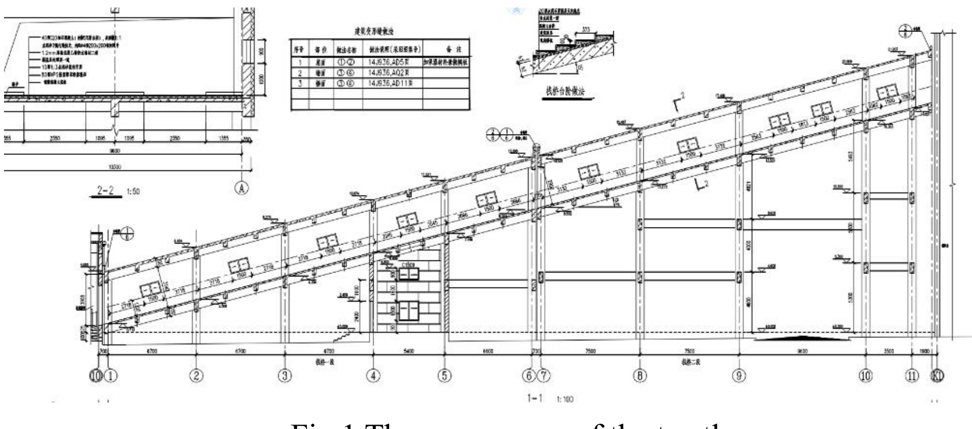

Fig.1 The appearance of the trestle

Frame column		Framed girder		floors
Size(mm)	amount	Size(mm)	amount	Thickness(mm)
550×650	2	300×800	11	120
600×700	20	300×600	10	
		300×700	26	

Table.1 Component information table

Note: All structural components utilize C30 concrete with HRB400 reinforcement as the primary steel bars.

3.2 Response testing

Within the boiler house of the facility, a reciprocating grate system is installed for fuel feeding. This equipment generates vibrations at 35-36 Hz during its operation cycle, which occurs every 360-400 seconds with a duration of approximately 6-9 seconds per cycle. Vibration signals from the structure were acquired using the 941B vibrometer manufactured by the Institute of Engineering Mechanics, China Earthquake Institute, with a sampling frequency of 250Hz. An 941B amplifier was used for amplification, the data acquisition instrument used was the DASP model from Beijing Orient Vibration and Noise Technology Research Institute. The test condition selected a measurement point between axis 8 and axis 9 of the material feeding bridge. The response information in three directions was collected, and one horizontal direction response signal was selected (Figure 2).

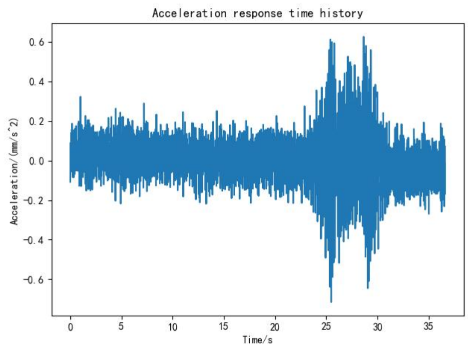

Fig.2 The Acceleration response time history of horizontal direction

4. Identification result

The Eigensystem Realization Algorithm (ERA), through continuous refinement and intrinsic algorithmic extensions, has evolved into a versatile framework for modal identification. Its integration with complementary methodologies has yielded novel research outcomes in structural dynamics characterization. For example, Juang, J. N [6] proposed the ERA/DC algorithm based on data correlation in 1988, the algorithm can more accurately identify noise in the response signal, thereby improving the accuracy of modal identification under environmental excitation. Li Lei-hong [7]replaced the input vector $u(k)$ in the state-space equation with a scalar F, where F represents a constant force. This extension of the ERA algorithm applies to systems responding under constant force excitation, Qi Quanquan, Juang, J. N[8],[9],[10],[11] introduced the observed Markov parameters and proposed an Extended Eigen-System Realization Algorithm (EERA), which expands the ERA algorithm to systems under random load excitation.

The material feeding Bridges is subjected random environmental excitation. As shown in Figure 2, the acceleration response signal of the bridge contains a decaying component of the free vibration response, which meets the conditions for using the ERA algorithm. To obtain the free vibration response signal, wavelet denoising technology[12] was applied to filter out the noise signal. Then, the decaying signal from 30 to 33 seconds was selected, and the ERA algorithm was used to identify the modal parameters. The discriminant index MAC was calculated, and the first six natural frequencies were compared with the results from finite element (FEM) analysis, as shown in Table 2.

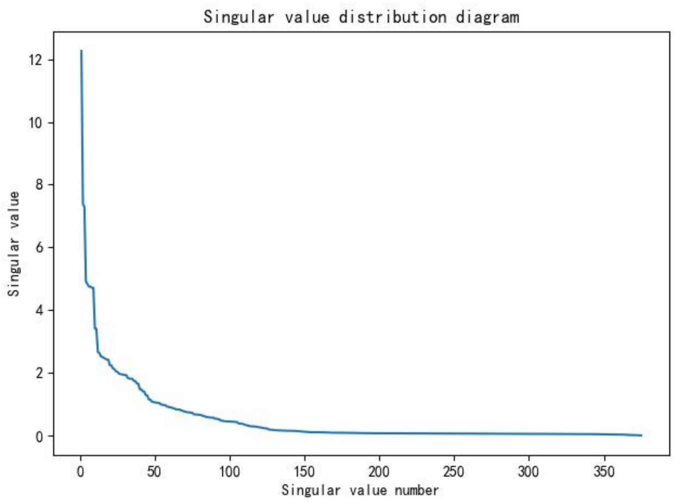

Fig.3 The Singular value of Hankel matrix distribution diagram

Order	Frequency/Hz		MAC	Error
	ERA	FEM		
1	0.8251	0.7886	0.9882	4.62%
2	1.1967	1.2878	0.9295	7.08%
3	1.3422	1.3704	0.9857	2.06%
4	2.4319	2.6532	0.9573	8.34%
5	4.8673	5.0327	0.9536	3.29%
6	6.8165	6.3654	0.9755	7.09%

Table.2 Comparison table of frequency recognition results

According to the data in the table, for structures under working conditions subjected to environmental excitation, the ERA algorithm demonstrates good accuracy in modal parameter identification. In this case, the maximum identification error was 8.34%. Considering that the feeding stacker bridge was subjected to random excitation with a duration of 360–400 seconds, the ability to extract the free vibration response portion of the signal greatly contributed to improving the identification accuracy. The difference between the ERA identified frequencies and the finite element analysis results may stem from the fact that the finite element model did not account for the decoration and material mass of the stacker bridge, as well as the element mesh division.

5. Conclusion

This paper uses the ERA method to identify the natural frequencies of the feeding stacker bridge at a biomass power plant. The results show that this modal identification method has good accuracy in practical engineering applications, and also highlights some issues that arise when applying the ERA method.

The accuracy of the ERA modal identification technique is highly subjective. The ERA algorithm relies on Singular Value Decomposition (SVD) of the Hankel matrix to eliminate noise and false modes. Both the order of the Hankel matrix and the rank of the system matrix after SVD are determined manually, which requires a certain level of experience.

How to remove noise from the system's response signal under environmental excitation to obtain the free vibration signal of the structure, and combining different signal processing methods with the ERA identification technique, is also a very worthwhile area of research.

This paper only verifies the accuracy of ERA in practical engineering applications. It is recommended that in future engineering practices, the algorithm parameters should be dynamically adjusted based on the sensor layout density/sampling frequency threshold, and an error-efficiency-cost multi-objective optimization model should be established to maximize the technical value.

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